Answers

1 Consumer arithmetic

Exercise 1.1

1 **a** \$14, **b** \$240, **c** \$78, **d** \$180, **e** \$182, **f** \$264, **g** \$673.75, **h** \$688.05 **2** **a** \$1770, **b** \$3808 **3** **a** \$420, **b** \$102.60, **c** \$82.23, **d** \$988.91 **4** **a** \$16, **b** \$15, **c** \$68.25, **d** \$181.13 **5** **a** \$903.60, **b** \$1245.50, **c** \$1761.20, **d** \$2736.32 **6** **a** \$72.44, **b** \$308.54, **c** \$21.17, **d** \$364.08 **7** **a** \$496.80, **b** \$1085.40 **8** **a** \$3240, **b** \$12 240 **9** **a** \$560, **b** \$644, **c** \$144 **10** \$2035.24 **11** \$720 **12** \$600 **13** 7 years **14** 9 months **15** 3% p.a. **16** 6.5% p.a. **17** **a** 24% p.a., **b** 12% p.a., **c** 8% p.a., **d** 2% p.a. **18** 5 months **19** 183 days

Exercise 1.2

1 **a**

Year	Opening balance	Interest	Closing balance
1st	\$10 000	\$10 000 × 0.06	\$10 600
2nd	\$10 600	\$10 600 × 0.06	\$11 236
3rd	\$11 236	\$11 236 × 0.06	\$11 910.16

b \$1910.16
2 **a** Balance = \$926.10; Interest = \$126.10, **b** Balance = \$2142.45; Interest = \$142.45
3 **a** \$515.15, **b** \$30.08
4 **a** \$1102.50, **b** \$843.65, **c** \$1739.69, **d** \$2652.95, **e** \$3800.31, **f** \$18 463.49, **g** \$12 452.56, **h** \$26 862.72
5 **a** \$1941.02, **b** \$3321.96, **c** 5346.24, **d** \$6967.56, **e** \$14 355.97, **f** \$9779.43, **g** \$29 866.15, **h** \$132 885.68
6 **a** \$5507.32, **b** \$5578.47, **c** \$5616.47, **d** \$5642.79 **7** **a** \$163.20, **b** \$1469.90, **c** \$2155.92, **d** \$329.73, **e** \$1920.74, **f** \$568.00, **g** \$2427.61, **h** \$2424.25, **i** \$15 432.27 **8** **a** \$10 790.80, **b** \$2790.80 **9** \$83 695.11
10 **a** \$251 001.90, **b** \$1001.90 **11** Christine, \$30.80 **12** B **13** A, \$1171.10 **14** \$30 216.03
15 Value = \$62 071.73; Interest = \$22 071.73 **16** **a** \$974 242, **b** \$1 261 672, **c** \$1 941 238 **17** **a** \$1.00, **b** \$2.40, **c** \$3.20, **d** \$56.40 **18** 9839 **19** 30 719 271 **20** \$42 370 **21** \$24 113 **22** \$600
23 **a** \$450, **b** \$1260, **c** \$1500, **d** \$2475

Exercise 1.3

1 \$15 360 **2** **a** \$2916, **b** \$1619, **c** \$2225, **d** \$1768 **3** \$6675 **4** \$768 **5** 6404 **6** 542
7 \$645 **8** **a** \$314, **b** \$436 **9** **a** 40 854 kL, **b** 9146 kL **10** 3794 **11** **a** \$343 490, **b** \$56 510
12 \$2955 **13** 5 years **14** 6 years **15** 7 years **16** \$15 250 **17** \$20 400 **18** 13%
19 **a** 9%, **b** \$750 000

Exercise 1.4

1 \$34.50 **2** 12 **3** **a** \$21.75, **b** \$24.65 **4** **a** \$240, **b** \$16 **5** **a** no, **b** \$565.25 **6** \$892.40
7 **a** \$2040, **b** \$240 **8** **a** \$60, **b** 13.3% **9** \$230 **10** B, \$502 **11** **a** \$4050, **b** \$19 050, **c** \$529.17
12 **a** \$11 495, **b** \$478.96 **13** **a** \$1380, **b** \$7820, **c** \$2815.20, **d** \$10 635.20, **e** \$295.42 **14** **a** \$2200, **b** \$8800, **c** \$3379.20, **d** \$12 179.20, **e** \$253.73 **15** **a** \$7056, **b** \$1456, **c** 13% **16** 9.52% **17** 19.5%

Exercise 1.5

1 **a** April, **b** 14, **c** 0.0593%, **d** \$3000, **e** \$930, **f** \$165.25, **g** \$212.60, **h** \$2787.40, **i** \$15, **j** \$0.63
2 **a** 13.14% p.a., **b** 16.64% p.a., **c** 20.25% p.a., **d** 7.80% p.a., **e** 8.46% p.a., **f** 10.19% p.a. **3** **a** \$0.48, **b** \$1.42, **c** \$7.78, **d** \$32.37 **4** **a** \$1.73, **b** \$13.04, **c** \$12.51, **d** \$109.07 **5** \$1.56 **6** **a** \$28.59, **b** \$898.59
7 **a** 0.042%, **b** 22, **c** \$320.94 **8** **a** \$1.16, **b** \$0.33, **c** \$169.49 **9** **a** \$0, **b** \$4.56 **10** **a** Yes, by \$430, **b** \$26.76 **11** \$10.30

ANSWERS

Exercise 1.6

1 **a** \$88.38, **b** \$87.68, **c** \$46.14, **d** \$26.09, **e** \$32.50, **f** \$22.24 **2** **a** \$263.76, **b** \$319.76, **c** \$263.88, **d** \$1315.20, **e** \$117.68, **f** \$163.84 **3** **a** **i** \$1170, **ii** \$170, **b** **i** \$5973.60, **ii** \$973.60, **c** **i** \$26 088, **ii** \$6088, **d** **i** \$39 538.80, **ii** \$4538.80 **4** **a** \$1582, **b** \$284 760, **c** \$84 760, **d** 42.38% **5** **a** \$554 400, **b** \$274 400, **c** 3.92% p.a. **6** **a** \$385 840, **b** 110.24%, **c** 3.67% p.a. **7** **a** Sunshine Bank, \$52 200 **8** \$6318 **9** \$31 524 **10** **a** \$320 000, **b** \$252 928, **c** 3.95% p.a. **11** **a** No, a month is longer than 2 fortnights. **b** $8\frac{1}{2}$ years, **c** \$83 844, **d** The loan is paid off more quickly and the amount of interest paid is reduced.

12 **a** **i** \$249 750, **ii** \$249 498.75, **iii** \$249 246.24, **iv** \$248 992.47, **b** Yes, the repayment is greater than the monthly interest. **c** No, the repayment is less than the monthly interest. **13** **a** \$1528.41, **b** Increase the size of the repayments.

Chapter 1 Review

1 **a** \$30, **b** \$118.13, **c** \$389.36, **d** \$313.50 **2** \$2480 **3** **a** \$28, **b** \$40.95, **c** \$45.31 **4** **a** \$770.04, **b** \$170.04 **5** \$800 **6** 7% p.a. **7** **a** \$21 000, **b** \$22 050, **c** \$23 152.50 **8** Balance = \$59 625.93, Interest = \$9625.93 **9** **a** \$2163.20, \$163.20, **b** \$1893.72, \$393.72, **c** \$5638.71, \$1888.71, **d** \$19 738.40, \$3738.40, **e** \$34 471.07, \$9471.07, **f** \$67 453.28, \$27 453.28 **10** **a** \$13 217.56, **b** \$13 431.36, **c** \$13 545.83, **d** \$13 625.23 **11** **a** \$4415.25, **b** \$7761.34, **c** \$11 716.59, **d** \$37 930.55, **e** \$19 046.02, **f** \$35 005.73 **12** **a** \$46 619.40, **b** 7.77% p.a. **13** B, by \$7.34 **14** A, by \$115.58 **15** \$634 443 **16** \$4.36 **17** **a** \$800, **b** \$5400, **c** \$15 000 **18** **a** \$18 000, **b** \$16 200, **c** \$11 810, **d** \$6974 **19** \$35 584 **20** **a** \$567 000, **b** \$486 902 **21** 14 years **22** **a** \$684, **b** no **23** **a** \$1085, **b** \$105, **c** 12.5% p.a. **24** **a** \$7920, **b** \$25 920, **c** \$540 **25** **a** \$3600, **b** \$8400, **c** \$2520, **d** \$303.33 **26** **a** \$6960, **b** \$960, **c** 8% p.a. **27** **a** \$3.52, **b** \$15.26 **28** \$5.45 **29** **a** \$88.15, **b** \$222.60, **c** \$215.46, **d** \$687.35 **30** **a** \$49.40, **b** \$648.40, **c** \$1411 **31** **a** \$2516, **b** \$754 800, **c** \$354 800, **d** 3.548% p.a. **32** \$19 440 **33** **a** \$240 000, **b** \$236 928, **c** \$3.3% p.a.

2 Trigonometry

Exercise 2.1

1 **a** opp = QR, adj = PQ, hyp = PR, **b** opp = EG, adj = FG, hyp = EF, **c** opp = XZ, adj = XY, hyp = YZ

2 **a** **i** $\frac{4}{5}$, **ii** $\frac{3}{5}$, **iii** $\frac{4}{3}$, **b** **i** $\frac{15}{17}$, **ii** $\frac{8}{17}$, **iii** $\frac{15}{8}$, **c** **i** $\frac{12}{13}$, **ii** $\frac{5}{13}$, **iii** $\frac{12}{5}$, **d** **i** $\frac{20}{29}$, **ii** $\frac{21}{29}$, **iii** $\frac{20}{21}$, **e** **i** $\frac{7}{25}$, **ii** $\frac{24}{25}$, **iii** $\frac{7}{24}$, **f** **i** $\frac{16}{65}$, **ii** $\frac{63}{65}$, **iii** $\frac{16}{63}$ **3** **a** α, **b** θ, **c** θ, **d** θ, **e** α, **f** α **4** **a** $\frac{15}{17}$, **b** $\frac{15}{20}$, **c** $\frac{15}{25}$, **d** $\frac{8}{17}$, **e** $\frac{8}{15}$, **f** $\frac{8}{17}$, **g** $\frac{15}{25}$, **h** $\frac{20}{15}$, **i** $\frac{15}{17}$, **j** $\frac{20}{25}$, **k** $\frac{15}{8}$, **l** $\frac{20}{25}$ **5** **a** $x = 3$, **b** $y = 9$, **c** $p = 20$, **d** $c = 27$, **e** $e = 60$, **f** $n = 35$ **6** **a** $\sin\theta = \frac{36}{85}$, $\cos\theta = \frac{77}{85}$, $\tan\theta = \frac{36}{77}$ **7** **a** 65 cm, **b** $\sin\theta = \frac{63}{65}$, $\cos\theta = \frac{16}{65}$, $\tan\theta = \frac{63}{16}$ **8** **a** 80 mm, **b** $\sin\alpha = \frac{80}{89}$, $\cos\alpha = \frac{39}{89}$, $\tan\alpha = \frac{80}{39}$ **9** **a** $\sin\theta = \frac{20}{29}$, $\cos\theta = \frac{21}{29}$, **b** $\cos\theta = \frac{7}{25}$, $\tan\theta = \frac{24}{7}$, **c** $\tan\theta = \frac{48}{55}$, $\sin\theta = \frac{48}{73}$ **10** $\sin\theta = \frac{x}{\sqrt{x^2 + y^2}}$, $\cos\theta = \frac{y}{\sqrt{x^2 + y^2}}$

Exercise 2.2

1 **a** 20°, **b** 36°, **c** 21°, **d** 71°, **e** 63°, **f** 84°, **g** 110°, **h** 138° **2** **a** 7°52′, **b** 22°6′, **c** 14°26′, **d** 76°33′, **e** 50°16′, **f** 68°5′, **g** 119°13′ **h** 164°2′ **3** **a** 0.80, **b** 0.11, **c** 0.37, **d** 0.72, **e** 7.57, **f** 24.73, **g** 19.31, **h** 16.03, **i** 40.38, **j** 313.76, **k** 1.40, **l** 0.31 **4** **a** 0.9134, **b** 12.85, **c** 25.30 **5** **a** 1.679, **b** 1.582 **6** **a** 20°11′, **b** 44°10′, **c** 7°10′, **d** 74°34′, **e** 74°36′, **f** 5°8′, **g** 35°54′, **h** 54°38′, **i** 0°49′, **j** 70°17′, **k** 52°14′, **l** 24°53′ **7** **a** 21°48′, **b** 64°9′, **c** 41°25′, **d** 33°3′ **8** **a** 0.7701, 0.6101, **b** 0.9711, 0.2388 **9** **a** 19°28′, **b** 59°45′, **c** 77°10′

Exercise 2.3

1 **a** $p = 5.5$, **b** $m = 5.7$, **c** $x = 35.2$, **d** $y = 48.6$, **e** $t = 1.6$, **f** $a = 58.0$, **g** $g = 50.5$, **h** $b = 13.1$, **i** $k = 38.6$, **j** $z = 33.4$, **k** $c = 15.3$, **l** $w = 54.3$, **m** $e = 32.5$, **n** $n = 17.8$, **o** $u = 87.9$ **2** **a** $x = 14.52$, **b** $t = 65.22$, **c** $e = 61.30$, **d** $m = 159.64$, **e** $c = 148.42$, **f** $v = 42.32$ **3** **a** 14.6 cm, **b** 16.56 m, **c** 10.7 cm, **d** 115 cm **4** 757 m

5 6.4 m **6** 7.5 m **7** 18 m **8** 195 cm **9** 11.4 m **10** 4.5 cm **11** **a** 10.5 cm, **b** 273 cm^2
12 **a** 25 cm, **b** 24 cm **13** **a** 7.5 cm, **b** 16.8 cm **14** 349 m **15** 20.2 m **16** 6.65 m **17** 1871 m
18 8.18 m **19** 50.3 m **20** 57 m **21** **a** 15 000 m, **b** 9000 m **22** **a** 44 mm, **b** 136.8 mm, **c** 240 mm,
d 80 mm **23** 55.59 cm **24** **a** 23.6 cm, **b** 31.2 cm,, **c** 368.16 cm^2 **25** **a** 10 cm, **b** 20 cm **26** 125 m
27 **a** $AD = 4.8$ cm, $DC = 11.2$ cm, **b** 16 cm **28** **a** 16.9 cm, **b** 69.9 cm **29** 115 m

Exercise 2.4

1 **a** 32°14′, **b** 40°14′, **c** 63°15′, **d** 77°54′, **e** 57°8′, **f** 19°11′, **g** 25°37′, **h** 67°19′, **i** 78°28′, **j** 50°39′, **k** 21°17′,
l 52°25′ **2** **a** 33°27′, **b** 53°41′, **c** 52°23′, **d** 73°55′, **e** 70°17′, **f** 41°54′, **g** 83°17′, **h** 24°59′, **i** 43°29′
3 **a** 41°49′, **b** 31°36′, **c** 35°6′ **4** **a** 62°6′, **b** 61°27′, **c** 36°59′ **5** 38°3′ **6** 18°40′ **7** 65°33′
8 32°43′ **9** 54°3′ **10** 70°26′ **11** 45°34′ **12** **a** 77°29′, **b** 35°24′ **13** 14°56′ **14** 51°23′
15 **a** 52°36′, **b** 42°9′, **c** 100°57′, **d** 68°6′ **16** 41°20′ **17** 68°35′ **18** 9°28′ **19** 12°32′ **20** 126°52′
21 **a** 36°16′, **b** 21°39′, **c** 71°23′, **d** 36°52′

Exercise 2.5

2 **a** $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$, $\tan\theta = \frac{3}{4}$, **b** $\sin\theta = \frac{12}{13}$, $\cos\theta = \frac{5}{13}$, $\tan\theta = \frac{12}{5}$, **c** $\sin\theta = \frac{12}{37}$, $\cos\theta = \frac{35}{37}$, $\tan\theta = \frac{12}{35}$

3 **a** 28°4′, **b** 33°8′, **c** 52°17′, **d** 61°52′ **5** **a** $\frac{24}{25}$, **b** $\frac{9}{41}$, **c** $\frac{21}{29}$, **d** $\frac{16}{65}$ **6** 45° **7** **a** 63°26′, **b** 18°26′,
c 58°0′, **d** 61°42′, **e** 58°39′, **f** 36°50′ **8** **a** 82°52′, **b** 36°52′, **c** 41°49′

Exercise 2.6

1 **a** **i** $\frac{a}{c}$, **ii** $\frac{b}{c}$, **iii** $\frac{b}{a}$, **iv** $\frac{a}{c}$, **b** **i** $\sin\theta = \cos(90° - \theta)$, **ii** $\cos\theta = \sin(90° - \theta)$ **3** **a** 90°, **c** Yes, $\sin\theta = \cos(90° - \theta)$.
4 **a** $\sin 30° = 0.5$, **b** $\cos 50° = 0.643$, **c** $\sin 75° = 0.966$, **d** $\cos 18° = 0.951$ **5** **a** $x = 40$, **b** $x = 55$, **c** $x = 25$,
d $x = 78$, **e** $x = 42$, **f** $x = 37$, **g** $x = 73$, **h** $x = 6$, **i** $x = 85$, **j** $x = 26$, **k** $x = 19$, **l** $x = 61$ **6** **a** 1, **b** 1, **c** 2, **d** $\frac{3}{4}$
7 **a** $x = 40$, **b** $x = 45$, **c** $x = 83$, **d** $x = 80$, **e** $x = 27$, **f** $x = 16$, **g** $x = 240$, **h** $x = 100$, **i** $x = 10$, **j** $x = 45$, **k** $x = 30$,
l $x = 40$ **8** **a** 0.52, **b** 0.21 **9** **a** 1, **b** $\tan\theta$, **c** 1, **d** $\tan\theta$, **e** $\sin^2\theta$, **f** $\cos^2\theta$, **g** $\sin\theta$, **h** $\cos\theta$, **i** $\cos^2\theta$

Exercise 2.7

1 **a** $\frac{1}{2}$, **b** $\frac{\sqrt{3}}{2}$, **c** $\frac{1}{\sqrt{3}}$, **d** $\frac{1}{\sqrt{2}}$, **e** $\frac{1}{\sqrt{2}}$, **f** 1, **g** $\frac{\sqrt{3}}{2}$, **h** $\frac{1}{2}$, **i** $\sqrt{3}$ **2** **a** 1, **b** $\frac{1}{4}$, **c** 3, **d** $\frac{1}{2}$, **e** $\frac{1}{3}$, **f** $\frac{3}{4}$ **3** **a** $\frac{1}{4}$,
b $\frac{1}{2}$, **c** 1, **d** $\frac{3}{4}$, **e** $\frac{\sqrt{6}}{4}$, **f** $\frac{\sqrt{6}}{2}$, **g** $\frac{\sqrt{6}}{6}$, **h** $\frac{\sqrt{2}}{4}$, **i** $\frac{1}{2}$, **j** $\frac{1}{4}$, **k** $1\frac{1}{2}$, **l** $\frac{\sqrt{3}}{4}$ **4** **a** 1, **b** $\frac{4\sqrt{3}}{3}$, **c** $\sqrt{2}$, **d** $\sqrt{3}$,
e $\frac{1+\sqrt{2}}{2}$, **f** $\frac{\sqrt{3}+\sqrt{2}}{2}$, **g** $\frac{3\sqrt{3}}{2}$, **h** $\frac{2+\sqrt{3}}{2}$, **i** $1\frac{1}{2}$ **5** **a** $\frac{3+5\sqrt{3}}{6}$, **b** $\frac{3\sqrt{2}+5\sqrt{3}}{6}$, **c** $\frac{3+4\sqrt{3}}{3}$,
d $\frac{1+\sqrt{2}+2\sqrt{3}}{2}$ **6** **a** 0, **b** $-\frac{1}{2}$, **c** 2, **d** $\frac{1}{2}$, **e** 4, **f** $1\frac{1}{2}$ **7** **a** 1, **b** $\sqrt{3}$, **c** $1\frac{1}{2}$, **d** $\sqrt{3}$, **e** 3, **f** $\frac{2\sqrt{3}}{3}$, **g** $\frac{\sqrt{6}}{2}$,
h $\frac{\sqrt{6}}{3}$ **9** **a** 30°, **b** 60°, **c** 45°, **d** 60°, **e** 60°, **f** 30°, **g** 30°, **h** 45°, **i** 45° **10** **a** $x = 7$, **b** $k = 5$, **c** $t = 4$,
d $n = 12$, **e** $u = 2\sqrt{3}$, **f** $e = 9$ **11** **a** $h = 14$, **b** $p = 17$, **c** $w = 12$ **12** **a** 45°, **b** 60°, **c** 30°, **d** 45°, **e** 60°,
f 60° **13** **a** $2\sqrt{3}$ cm, **b** $4\sqrt{3}$ cm^2 **14** **a** $x = 8$, **b** $y = 13$, **c** 72 cm^2 **15** **a** 30 cm, **b** $150\sqrt{3}$ cm^2
16 $80\sqrt{3}$ m

Exercise 2.8

1 **a** **i** A = N50°E, B = S15°E, C = N66°W, **ii** $A = 050°$, $B = 165°$, $C = 294°$, **b** **i** A = S74°E, B = S61°W, C = N9°W,
ii $A = 106°$, $B = 241°$, $C = 351°$, **c** **i** A = N72°E, B = S33°W, C = N44°W, **ii** $A = 072°$, $B = 213°$, $C = 316°$
2 **a** 210°, **b** 055°, **c** 328°, **d** 123° **3** **a** 9°, **b** 137° **4** **a** 48.9 km, **b** 64.9 km, **c** 123.3 km, **d** 91.7 km
5 **a** 90.3 km, **b** 8.41 km, **c** 23.2 km, **d** 16.7 km **6** **a** **i** 139°, **ii** 319°, **b** **i** 314°, **ii** 134°, **c** **i** 219°, **ii** 039°,
d **i** 041°, **ii** 221° **7** **a** **i** $\angle HSP = 43° + 47° = 90°$, **ii** 218 m, **b** **i** $\angle HBS = 16° + 74° = 90°$, **ii** 2.6 km
8 **a** 228 km, **b** 038° **9** **a** 28.36 NM, **b** 4.02 NM, **c** 29 NM, **d** 098° **10** 123 km

ANSWERS

Chapter 2 Review

1 **a** $\frac{7}{25}$, **b** $\frac{18}{30}$, **c** $\frac{24}{7}$, **d** $\frac{24}{30}$, **e** $\frac{18}{24}$, **f** $\frac{24}{25}$ **2** $x = 15$ **3** $\sin\theta = \frac{65}{97}, \cos\theta = \frac{72}{97}$ **4** 54.15
5 **a** 74°48′, **b** 22°24′ **6** 0.7315 **7** **a** $a = 29.9$, **b** $p = 31.0$, **c** $z = 22.1$ **8** **a** 26.4 cm, **b** 25.1 cm
9 **a** 65°33,′ **b** 70°44′, **c** 81°33′ **10** **a** 62.2 cm, **b** 128.1 cm, **c** 62°14′ **11** 44 m **12** 6.05 m
13 2618 m **14** 53°26′ **15** 52°28′ **16** **a** 38 cm, **b** 98 cm **17** 45°14′ **18** C **19** **a** $\sin\theta$, **b** $\cos\theta$
20 **a** 75°58′, **b** 26°34′, **c** 59°2′ **21** **a** $x = 20$, **b** $x = 65$, **c** $x = 16$, **d** $x = 73$, **e** $x = 24$, **f** $x = 45$ **22** **a** 1, **b** $\tan\theta$,
c $\cos^2\theta$, **d** $\sin\theta$ **23** **a** $\frac{\sqrt{2}}{4}$, **b** $\frac{3}{4}$, **c** $\sqrt{2}$, **d** $\frac{5\sqrt{3}}{6}$, **e** $\frac{1}{2}$, **f** $4\frac{1}{2}$, **g** 2, **h** $\frac{\sqrt{6}}{3}$ **24** **a** $x = 6$, **b** $x = 4\sqrt{3}$
25 45° **26** **a** **i** N9°E, **ii** 009°, **b** **i** S55°E, **ii** 125°, **c** **i** S77°W, **ii** 257°, **d** **i** N42°W, **ii** 318°
27 **a** 292°, **b** 057° **28** 18° **29** **a** 92.2 km, **b** 7.9 km **30** **a** 134.8 NM, **b** 15.29 km **31** **a** **i** 122°,
ii 302°, **b** **i** 331°, **ii** 151° **32** **a** $\angle IJK = 43° + 47° = 90°$, **b** 268 m **33** **a** $\angle PQR = 59° + 31° = 90°$, **b** 56 km

3 Volume and surface area

Exercise 3.1

1 **a** 24 cm^2, **b** 54 cm^2, **c** 174.96 cm^2, **d** 564.54 cm^2 **2** **a** 100 cm^2, **b** 312 cm^2, **c** 176 cm^2 **3** 224.54 cm^2
4 **a** 120 cm^2, **b** 600 cm^2, **c** 372.95 cm^2 **5** **a** 392 cm^2, **b** 1540 cm^2 **6** **a** 320 cm^2, **b** 840 cm^2
7 **a** **i** $x = 19.5$, **ii** 1260 cm^2, **b** **i** $h = 24$, **ii** 4944 cm^2 **8** **a** 4 cm, **b** 7 cm, **c** 13 cm **9** **a** $y = 17$, **b** 820 cm^2
10 **a** 137.9 m^2, **b** \$1991.10 **11** **a** 72.2 m^2, **b** \$8.50 **12** 8 cm, 6 cm, 5 cm **13** 9 cm **14** 12 cm
15 **a** 486 cm^2, **b** 1028 cm^2, **c** 1406 cm^2 **d** 866 cm^2

Exercise 3.2

1 **a** 105 cm^2, **b** 242 cm^2 **2** **a** 340 cm^2, **b** 216 cm^2, **c** 338 cm^2 **3** **b** $36\sqrt{3}$ cm^2 **4** **a** 25 cm, **b** 30 cm,
c 1356 cm^2 **5** **a** 74.2 cm^2, **b** 254.1 cm^2, **c** 535.5 cm^2 **6** **a** 266.4 cm^2, **b** 970.3 cm^2 **7** **a** 1119.1 cm^2,
b 662.7 cm^2 **8** **b** $9\sqrt{2}$ cm, **c** 507.8 cm^2

Exercise 3.3

1 **a** 483.8 cm^2, **b** 446.2 cm^2, **c** 301.6 cm^2, **d** 1297.5 cm^2 **2** **a** 80π cm^2, **b** 198π cm^2 **3** **a** 150.8 cm^2,
b 339.3 cm^2, **c** 1979.2 cm^2, **d** 706.9 cm^2, **e** 1653.2 cm^2, **f** 181.8 cm^2 **4** **a** 19.1 cm, **b** 85.1 cm, **c** 8 cm
5 **a** 400 cm^2, **b** 64π cm^2, **c** 200π cm^2, **d** 1229.4 cm^2 **6** **a** 673.6 cm^2, **b** 298.0 cm^2 **7** **a** $A = \pi r^2 + 2\pi rh$,
b 4134 mm^2 **8** **a** 179 cm^2, **b** 456 cm^2 **9** **a** 73.42 m^2, **b** \$216.60 **10** **a** 9π m^2, **b** 8π m^2, **c** 0.34π m^2,
d 54.5 m^2 **11** \$1430.68 **12** **a** 922 cm^2, **b** 107 cm^2, **c** 1126 cm^2, **d** 982 cm^2 **13** 7 cm **14** 234π cm^2

Exercise 3.4

1 **a** 65π cm^2, **b** 168π cm^2, **c** 60π cm^2 **2** **a** 373.8 cm^2, **b** 188.5 cm^2, **c** 221.7 cm^2, **d** 537.8 cm^2, **e** 975.7 cm^2,
f 221.6 cm^2 **3** **a** 616 cm^2, **b** 1257 cm^2, **c** 845 cm^2, **d** 499 cm^2, **e** 302 cm^2, **f** 401 cm^2 **4** **a** 36π cm^2,
b 16π cm^2, **c** 144π cm^2, **d** 400π cm^2 **5** **a** 301.6 cm^2, **b** 433.2 cm^2, **c** 41.4 cm^2 **6** **a** 192π cm^2, **b** 27π cm^2
7 **a** 9 cm, **b** 11 cm, **c** 26 cm **8** **a** 2.82 cm, **b** 5.09 cm, **c** 13.08 cm **9** **a** $x = 39$, S.A. = 2545 cm^2,
b $x = \sqrt{250}$, S.A. = 702 cm^2, **c** $x = \sqrt{209.96}$, S.A. = 1616 cm^2 **10** 514 718 540 km^2 **11** **a** 15 cm,
b 679 cm^2 **12** **a** 25 cm, **b** 175π cm^2 **13** 4 cm **14** 3 : 4 **15** **a** 624 cm^2, **b** 591 cm^2, **c** 144 cm^2,
d 151 cm^2, **e** 837 cm^2, **f** 585 cm^2 **16** 349π cm^2

Exercise 3.5

1 **a** 24 units^3, **b** 30 units^3, **c** 90 units^3 **2** **a** 30 cm^3, **b** 52 cm^3, **c** 108 cm^3, **d** 93 cm^3, **e** 319.5 cm^3, **f** 223.1 cm^3
3 **a** 60 cm^3, **b** 112 cm^3, **c** 130 cm^3 **4** **a** 8 cm^3, **b** 125 cm^3, **c** 729 cm^3, **d** 1728 cm^3 **5** **a** 77.64 m^3,
b 1225.73 m^3, **c** 0.52 m^3 **6** **a** 8 cm, **b** 6.2 cm, **c** 9.2 cm, **d** 11.1 cm **7** **a** 3 m, **b** 9 m, **c** 3 m, **d** 12 m,
e 11.5 m, **f** 24.4 m **8** **a** 54 cm^3, **b** 176 cm^3, **c** 450 cm^3 **9** **a** 195 cm^3, **b** 432 mm^3 **10** **a** 225 mm^3,
b 51 m^3, **c** 206.08 cm^3 **11** **a** 168 cm^3, **b** 384 m^3, **c** 2478 mm^3 **12** **a** 9 cm, **b** 23 mm^2, **c** 7.5 cm, **d** 26.3 m^2
13 **a** **i** $x = 10.8$, **ii** 393.66 cm^3, **b** **i** $p = 6$, **ii** 28.38 cm^3, **c** **i** $k = 7$, **ii** 159.6 cm^3 **14** **a** 110 m^3, **b** 415 m^3,
c 2208 m^3, **d** 311.5 m^3, **e** 1137.6 m^3, **f** 651 m^3, **g** 862.2 m^3, **h** 795 m^3, **i** 7308 m^3 **15** **a** 486 cm^2, **b** 1331 cm^3
16 6 units

Exercise 3.6

1 **a** 628 cm^3, **b** 3308 cm^3, **c** 2898 cm^3, **d** 565 cm^3, **e** 1512 cm^3, **f** 352 cm^3 **2** **a** 502.7 cm^3, **b** 7926.7 cm^3, **c** 10 053.1 cm^3, **d** 24 119.3 cm^3 **3** **a** 20π mm^3, **b** 324π mm^3, **c** 54π mm^3, **d** 44π mm^3 **4** **a** 1891 cm^3, **b** 1196 cm^3, **c** 193.6 cm^3, **d** 732.4 cm^3, **e** 271.3 cm^3, **f** 538.6 cm^3 **5** **a** 12 m, **b** 15 m **6** **a** 11 m, **b** 16 m **7** **a** 8567 cm^3, **b** 6584 cm^3, **c** 4739 cm^3, **d** 1269 cm^3, **e** 294 cm^3, **f** 993 cm^3 **8** **a** 240π cm^3, **b** 960 cm^3, **c** 78.5% **9** **a** $h = 14$ cm, $r = 4$ cm, **b** 16 889 cm^3, **c** 16.9 L **10** 2.3 L **11** **a** 15.394 m^3, **b** 15 394 L **12** 16 cm **13** **a** 156π cm^2, **b** 845π cm^3

Exercise 3.7

1 **a** 50 cm^3, **b** 56 cm^3, **c** 132 cm^3, **d** 435 cm^3, **e** 421.08 cm^3, **f** 664.02 cm^3 **2** **a** 336 cm^3, **b** 360.8 cm^3, **c** 364 cm^3 **3** **a** 24.5 cm^3, **b** 105 m^3, **c** 176 mm^3, **d** 211.68 cm^3, **e** 782.04 m^3, **f** 67.6 mm^3 **4** 338.4 cm^3 **5** **a** 68 cm^2, **b** 272 cm^3 **6** **a** 6 cm, **b** 5 cm, **c** 5 cm, **d** 12 cm **7** **a** 270.48 cm^3, **b** 6.5 cm **8** **a** 20 cm^3, **b** 36 cm^3 **9** **a** 8 cm, **b** 15 cm, **c** 720 cm^3 **10** **a** 37.5 cm, **b** 1687.5 cm^2, **c** 34 875 cm^3 **11** **a** 640 cm^3, **b** 1004.4 cm^3, **c** 624 cm^3, **d** 7207.2 cm^3, **e** 1178.45 cm^3, **f** 805 cm^3 **12** 787.5 cm^3 **13** **a** 20 cm, **b** $x = 42$, **c** 8400 cm^3 **14** **a** $OA = 9$ cm, $OB = 16$ cm, **b** 2304 cm^3, **c** 288 cm^3, **d** $\frac{1}{8}$

Exercise 3.8

1 **a** 235.6 cm^3, **b** 737.2 cm^3, **c** 4021.2 cm^3, **d** 3167.8 cm^3, **e** 1862.7 cm^3, **f** 2907.2 cm^3 **2** **a** 186 cm^3, **b** 504 cm^3, **c** 351 cm^3, **d** 682 cm^3 **3** **a** 32π cm^3, **b** 240π cm^3, **c** 75π cm^3 **4** **a** **i** $x = 12$, **ii** 5277.9 cm^3, **b** **i** $x = 24$, **ii** 2513.3 cm^3, **c** **i** $x = 8$, **ii** 27.1 cm^3 **5** **a** 10 mm^3, **b** 75 mm^3 **6** **a** 11 mm, **b** 154 mm^2 **7** **a** **i** 36 cm, **ii** 8482 cm^3, **b** **i** 60 cm, **ii** 7603 cm^3 **8** **a** $\frac{2}{3}$, **b** 13 854 cm^3 **9** 125.7 cm^3 **10** **a** 26 cm, **b** 3716.5 cm^3 **11** **a** 235 cm^3, **b** 594 cm^3, **c** 1891 cm^3, **d** 1172 cm^3, **e** 558 cm^3, **f** 368 cm^3 **12** **a** **i** 7 cm, **ii** 9 cm, **b** 15 cm **13** **a** 5.2 L, **b** 1.6 L **14** 28.3 cm **15** 18.72 cm

Exercise 3.9

1 **a** 34 cm^3, **b** 268 cm^3, **c** 860 cm^3, **d** 524 cm^3, **e** 1437 cm^3, **f** 8181 cm^3 **2** **a** 1047.4 mm^3, **b** 20.6 mm^3 **c** 33.5 mm^3, **d** 448.9 mm^3 **3** **a** 36π cm^3, **b** 972π cm^3, **c** $12\,348\pi$ cm^3, **d** 2304π cm^3, **e** 4500π cm^3, **f** 7776π cm^3 **4** **a** 262 m^3, **b** 575 m^3, **c** 1740 m^3 **5** **a** 486 cm^3, **b** 318 cm^3, **c** 1334 cm^3, **d** 304 cm^3, **e** 16 cm^3, **f** 175 cm^3 **6** 524 g **7** **a** 972π cm^3, **b** 1458π cm^3, **c** 1526.8 cm^3 **8** **a** 32 cm, **b** 17 157 cm **9** 3 : 4 **10** **a** 3 cm, **b** 6 cm, **c** 15 cm **11** 6.2 cm **12** **a** 11 cm^3, **b** 11 mL **13** **a** $2r$ cm, **b** 5 cm, **c** $\frac{4}{3}\pi r^3$ cm^3, **d** 2 : 3, **e** 16π cm^3

Chapter 3 Review

1 **a** 184 cm^2, **b** 300 cm^2 **2** 14 cm **3** **a** $x = 15$, **b** 5430 cm^2 **4** 364 cm^2 **5** **a** $PE = \sqrt{221}$ cm, $PF = \sqrt{170}$ cm, **b** 749 cm^2 **6** 1004 cm^2 **7** 1175 cm^2 **8** 7 cm **9** 854.5 cm^2 **10** 358.8 cm^2 **11** 49.5 cm^2 **12** **a** 60π cm^2, **b** 267 cm^2 **13** 232.4 cm^2 **14** 3 cm **15** 7 cm **16** **a** 406.8 cm^2, **b** 417.0 cm^2 **17** 120 cm^3 **18** **a** 343 cm^3, **b** 105 cm^3 **19** **a** 780 cm^3, **b** 1700 cm^3, **c** 560 cm^3 **20** 16.5 mm^2 **21** **a** $w = 80$, **b** 72 000 cm^3, **c** 72 L **22** 6065.9 cm^3 **23** **a** 594 cm^3, **b** 24 cm^3 **24** **a** 320 cm^3, **b** 522.97 cm^3 **25** **a** 1990 cm^3, **b** 1960 cm^3, **c** 1437 cm^3, **d** 524 cm^3 **26** **a** 162π cm^3, **b** 75π cm^3, **c** 36π cm^3 **27** **a** 9 cm, **b** 1017.9 cm^3 **28** **a** 6 cm^3, **b** 21π cm^3 **29** **a** 5 cm, **b** 8 cm **30** a 4.8 cm, **b** 11 cm, **c** 12 cm **31** **a** 96 cm^2, **b** 3375 cm^3 **32** **a** 370 cm^3, **b** 811 cm^3, **c** 980 cm^3, **d** 709 cm^3

4 Deductive geometry

Exercise 4.1

1 **a** $x = 50$, **b** $y = 110$, **c** $t = 64$, **d** $u = 123$, **e** $p = 52$, **f** $k = 20$, **g** $m = 103$, **h** $a = 30$, **i** $z = 45$, **j** $r = 35$, **k** $w = 72$, **l** $n = 43$ **2** **a** $x = 100$, **b** $p = 52$, **c** $r = 145$, **d** $k = 39$, **e** $y = 76$, **f** $f = 47$ **3** **a** $x = 53$, **b** $p = 27$, **c** $c = 35$, **d** $a = 57$, **e** $t = 28$, **f** $h = 60$, **g** $q = 109$, **h** $k = 137$, **i** $s = 14$ **4** **a** $z = 153$, **b** $b = 62$, **c** $n = 26$, **d** $a = 70$, **e** $u = 5$, $v = 8$, **f** $x = 7$, $y = 25$, **g** $p = 90$, **h** $v = 3$, $w = 10$, **i** $k = 14$, **j** $d = 110$, $e = 8$, **k** $r = 90$, $s = 46$, **l** $j = 20$, $k = 6$, **m** $a = 90$, $b = 18$, **n** $f = 15$, **o** $y = 90$, $z = 4$ **5** **a** $x = 53$, $y = 53$, **b** $p = 66$, $q = 24$, **c** $a = 36$, $b = 72$, **d** $g = 26$, **e** $k = 36$, **f** $n = 5$, **g** $c = 16$, **h** $b = 29$, **i** $v = 9$ **6** **a** $a = 96$, $b = 84$, **b** $u = 49$, $v = 41$, **c** $c = 33$, $d = 147$,

ANSWERS

d $m = 36$, $n = 36$, **e** $j = 62$, $k = 118$, **f** $g = 118$, $h = 118$, $i = 62$ **7 a** $t = 61$, **b** $n = 44$, **c** $r = 55$, **d** $c = 53$, **e** $h = 112$, **f** $k = 63$ **8 a** yes, **b** no, **c** no **9 a** $a = 72$, $b = 108$, $c = 72$, **b** $x = 24$, $y = 54$, $z = 18$, **c** $p = 21$, $q = 30$, $r = 98$ **10 a** $b = 13$, **b** $e = 24$, **c** $g = 20$, **d** $x = 11$, **e** $t = 31$, **f** $n = 12$, **g** $s = 7$, **h** $w = 15$, **i** $y = 17$ **11 a** $a = 54$, $b = 55$, **b** $m = 109$, $n = 29$, **c** $p = 27$, $q = 27$, **d** $x = 127$, $y = 127$, **e** $c = 65$, $d = 115$, **f** $g = 59$, $h = 59$, **g** $r = 60$, $s = 148$, **h** $u = 69$, $v = 83$, **i** $j = 45$, $k = 135$, **j** $e = 103$, $f = 103$, **k** $p = 66$, $q = 105$, **l** $m = 112$, $n = 68$ **12 a** $a = 65$, $b = 54$, **b** $e = 27$, $f = 63$, **c** $p = 119$, $q = 30$, **d** $u = 29$, $v = 75$, **e** $g = 119$, $h = 90$, **f** $v = 57$, $w = 57$, **g** $j = 32$, $k = 32$, **h** $c = 121$, $d = 75$, **i** $s = 60$, $t = 123$ **13 a** $m = 38$, $n = 142$, **b** $b = 90$, $c = 137$, **c** $a = 90$, $b = 21$, $c = 21$, $d = 21$, **d** $a = 76$, $b = 76$, $c = 76$, **e** $x = 73$, $y = 73$, $z = 73$, **f** $x = 90$, $y = 45$, $z = 45$ **14** $x = 35$ **15** $x = 18$

Exercise 4.2

1 a 4, **b** 6, **c** 8, **d** 10, **e** 5, **f** 7, **g** 9, **h** 12, **i** 11 **2 a** convex, **b** non-convex, **c** convex, **d** non-convex **3 a** regular, **b** irregular, **c** irregular, **d** regular **4 a i** rectangle, **ii** rhombus, **b** no — all sides and all angles are equal in regular polygons. **5** 540° **6 a** 108°, **b** 120°, **c** 135°, **d** 144° **7** 162° **8** $c = 42$ **9 a** 60°, **b** 72°, **c** 30° **10** 120° **11** 15° **12 a** 8, **b** 10, **c** 18, **d** 9 **15 a** 540°, **b** 2340°

Exercise 4.3

1 a $\angle AED = 73°$ (co-interior $\angle$s, $AB \parallel CD$)
$\angle CEF = 73°$ (vertically opposite $\angle$s)
$\therefore x = 73$

b $\angle LKM = 64°$ ($\angle K$ bisects $\angle JKM$)
$\angle KMN = 64°$ (alternate $\angle$s, $KL \parallel NM$)
$\therefore x = 64$

c $\angle XZV = 44°$ (corresponding $\angle$s, $TU \parallel VW$)
$\angle VZS = 46°$ (adjacent $\angle$s in a right angle)
$\therefore x = 46$

d $\angle DIH = 67°$ (corresponding $\angle$s, $DE \parallel FG$)
$\angle DEJ = 67°$ (corresponding $\angle$s, $GH \parallel EJ$)
$\therefore x = 67$

e $\angle KLN = 111°$ (co-interior $\angle$s, $KL \parallel MN$)
$\angle KLJ = 153°$ ($\angle$s at a point)
$\angle IJL = 27°$ (co-interior $\angle$s, $IJ \parallel KL$)
$\therefore x = 27$

f $\angle TUX = 25°$ (adjacent $\angle$s on a straight line)
$\angle TUV = 48°$ (sum of adjacent $\angle$s)
$\angle STU = 48°$ (alternate $\angle$s, $ST \parallel UV$)
$\therefore x = 48$

g $\angle BFG = 58°$ (vertically opposite $\angle$s)
$\angle EGD = 122°$ (co-interior $\angle$s, $AB \parallel CD$)
$\angle HGD = 61°$ (HG bisects $\angle EGD$)
$\therefore x = 61$

h $\angle DBC = 73°$ (adjacent $\angle$s in a right angle)
$\angle BDE = 107°$ (co-interior $\angle$s, $DE \parallel BC$)
$\angle DEG = 107°$ (alternate $\angle$s, $BD \parallel EG$)
$\angle DEF = 103°$ ($\angle$s at a point)
$\therefore x = 103$

i $\angle WRS = 77°$ (corresponding $\angle$s, $PQ \parallel RS$)
$\angle TRP = 77°$ (WV bisects $\angle TRS$)
$\angle VRU = 77°$ (vertically opposite $\angle$s)
$\therefore x = 77$

2 a $\angle BCD = 47°$ (alternate $\angle$s, $AB \parallel CE$)
$\angle BDE = 137°$ (exterior $\angle$ of a Δ)
$\therefore x = 137$

b $\angle PRQ = 28°$ (vertically opposite $\angle$s)
$\angle PQR = 116°$ ($\angle$ sum of a Δ)
$\angle PQU = 58°$ (UQ bisects $\angle PQR$)
$\therefore x = 58$

c $\angle ZXY = 62°$ ($\angle$ sum of a Δ)
$\angle WXV = 62°$ (vertically opposite $\angle$s)
$\angle VWX = 62°$ (base $\angle$s of isosceles Δ, $VW = VX$)
$\angle WVX = 56°$ ($\angle$ sum of a Δ)
$\therefore x = 56$

d $\angle EFG = 71°$ (co-interior $\angle$s, $EF \parallel HG$)
$\angle DEF = 71°$ (alternate $\angle$s, $DE \parallel FG$)
$\angle EDF = 67°$ ($\angle$ sum of a Δ)
$\therefore x = 67$

e $\angle LMN = 48°$ (corresponding $\angle$s, $JK \parallel LM$)
$\angle LNO = 81°$ (exterior $\angle$ of a Δ)
$\angle ONP = 81°$ (OK bisects $\angle LNP$)
$\therefore x = 81$

f $\angle CBD = 61°$ (adjacent $\angle$s in a right angle)
$\angle CDB = 61°$ (base $\angle$s of an isosceles Δ, $CB = CD$)
$\angle DCE = 61°$ (alternate $\angle$s, $CE \parallel BD$)
$\angle CED = 61°$ (base $\angle$s of an isosceles Δ, $DC = DE$)
$\therefore x = 61$

g $\angle EDG = 83°$ ($\angle$s at a point)
$\angle EFG = 87°$ ($\angle$ sum of a quadrilateral)
$\angle GFH = 93°$ (adjacent $\angle$s on a straight line)
$\therefore x = 93$

h $\angle QRS = 72°$ (alternate $\angle$s, $PQ \parallel RS$)
$\angle UTS = 72°$ (opposite $\angle$s of a parallelogram)
$\angle STV = 18°$ (adjacent $\angle$s in a right angle)
$\therefore x = 18$

j $\angle UWV = 60°$ ($\angle$ in an equilateral Δ)
$\angle VWX = 65°$ ($\angle$s at a point)
$\angle WVY = 115°$ (co-interior $\angle$s, $VY \parallel WX$)
$\therefore x = 115$

l $\angle IML = 108°$ ($\angle$ in a regular pentagon)
$\angle MLN = 108°$ (alternate $\angle$s, $LN \parallel IM$)
$\angle LMN = 45°$ ($\angle$ sum of a Δ)
$\therefore x = 45$

b $PQ \parallel SR$ (opposite sides of a rectangle are parallel)
$\angle QPR = 21°$ (alternate $\angle$s, $PQ \parallel SR$)
$\therefore x = 21$

4 a $BC = CD$ (sides of a rhombus)
$\angle CBD = 17°$ (base $\angle$s of an isosceles Δ, $BC = CD$)
$\therefore m = 17$

c $AD \parallel BC$ (opposite sides of a rhombus are parallel)
$\angle ABC = 64°$ (co-interior $\angle$s, $AD \parallel BC$)
$\angle EBC = 32°$ (diagonals of a rhombus bisect $\angle$s at the vertices)
$\therefore m = 32$

e $\angle BEC = 90°$ (diagonals of a rhombus are perp.)
$\angle FEC = 61°$ (alternate $\angle$s, $EF \parallel DC$)
$\angle BEF = 29°$ (adjacent $\angle$s in a right angle)
$\therefore m = 29$

i $\angle VWX = 123°$ (co-interior $\angle$s, $UV \parallel XW$)
$\angle XWZ = 85°$ ($\angle$s at a point)
$\angle WXY = 151°$ ($\angle$ sum of a quadrilateral)
$\therefore x = 151$

k $OA = OB$ (equal radii)
$\angle OAB = 55°$ (corresponding $\angle$s, $AB \parallel CD$)
$\angle OBA = 55°$ (base $\angle$s of an isosceles Δ, $OA = OB$)
$\angle EOB = 110°$ (exterior $\angle$ of a Δ)
$\therefore x = 110$

3 a $\angle QRS = 90°$ ($\angle$ in a rectangle)
$\angle QSR = 37°$ ($\angle$ sum of a Δ)
$\therefore x = 37$

c $PT = TS$ (diagonals of a rectangle are equal and bisect each other)
$\angle PST = 67°$ (base $\angle$s of an isosceles Δ, $PT = TS$)
$\angle PTS = 46°$ ($\angle$ sum of a Δ)
$\angle QTR = 46°$ (vertically opposite $\angle$s)
$\therefore x = 46$

b $\angle AEB = 90°$ (diagonals of a rhombus are perp.)
$\angle ABE = 36°$ ($\angle$ sum of a Δ)
$\angle CBE = 36°$ (diagonals of a rhombus bisect $\angle$s at the vertices)
$\therefore m = 36$

d $\angle ADC = 52°$ (adjacent $\angle$s on a st. line)
$\angle ADB = 26°$ (diagonals of a rhombus bisect $\angle$s at the vertices)
$\therefore m = 26$

f $\angle DEF = 75°$ (base $\angle$s of an isosceles Δ, $DE = DF$)
$\angle EDF = 30°$ ($\angle$ sum of a Δ)
$AB \parallel DC$ (opposite sides of a rhombus are parallel)
$\angle DAB = 150°$ (co-interior $\angle$s, $AB \parallel DC$)
$\therefore m = 150$

Exercise 4.4

1 Let $\angle AEC = \alpha$
$\angle AED = 180° - \alpha$ (adjacent $\angle$s on a straight line)
$\angle DEB = 180° - (180° - \alpha) = \alpha$ (adjacent $\angle$s on a straight line)
$\therefore \angle AEC = \angle DEB$ (both equal to α)

3 Let $\angle POR = \alpha$
$\angle POS = \alpha$ (PO bisects $\angle ROS$)
$\angle ROQ = 180° - \alpha$ (adjacent $\angle$s on a straight line)
$\angle SOQ = 180° - \alpha$ (adjacent $\angle$s on a straight line)
$\therefore \angle ROQ = \angle SOQ$ (both equal to $180° - \alpha$)

2 a Let $\angle ABD = \alpha$
$\angle DBC = 90° - \alpha$ (adjacent $\angle$s in a right angle)
$\angle EBC = \alpha$ (adjacent $\angle$s in a right angle)
$\therefore \angle ABD = \angle EBC$ (both equal to α)

b $\angle ABE + \angle DBC = 90° + \alpha + 90° - \alpha$
$= 180°$

4 Let $\angle ABD = \alpha$ and $\angle FBC = \beta$
$\angle DBE = \alpha$ (DB bisects $\angle ABE$)
$\angle EBF = \beta$ (FB bisects $\angle EBC$)
$2x + 2\beta = 180°$ ($\angle$s on a straight line)
$\therefore \alpha + \beta = 90°$
$\therefore \angle DBF = \alpha + \beta$ (sum of adjacent $\angle$s)
$= 90°$
$\therefore DB \perp BF$

ANSWERS

5 Let $\angle TWX = \alpha$
$\angle TUY = \alpha$ (corresponding $\angle$s, $WX \parallel UY$)
$\angle YUV = \alpha$ (UY bisects $\angle TUV$)
$\angle UVZ = \alpha$ (alternate $\angle$s, $UY \parallel ZV$)
$\therefore \angle TWX = \angle UVZ$ (both equal to α)

6 Let $\angle ABC = \alpha$ and $\angle CDE = \beta$
Construct FC, parallel to BA and DE
$\angle BCF = \alpha$ (alternate $\angle$s, $BA \parallel FC$)
$\angle DCF = \beta$ (alternate $\angle$s, $FC \parallel DE$)
$\angle BCD = \alpha + \beta$ (sum of adjacent $\angle$s)
$\therefore \angle BCD = \angle ABC + \angle CDE$

7 Let $\angle AEF = \alpha$, $\angle CAE = \beta$ and $\angle ACD = \gamma$
$\angle BAE = 180° - \alpha$ (co-interior $\angle$s, $AB \parallel EF$)
$(180° - \alpha) + \beta + \gamma = 180°$ (co-interior $\angle$s, $AB \parallel CD$)
$\therefore \alpha = \beta + \gamma$

8 Let $\angle CHG = \alpha$
$\angle GHD = 180° - \alpha$ (adjacent $\angle$s on a straight line)
$\angle CDI = 180° - \alpha$ (alternate $\angle$s, $GH \parallel DJ$)
$\angle DIE = 180° - (180° - \alpha) = \alpha$ (co-interior $\angle$s, $CD \parallel EF$)
$\angle JIF = \alpha$ (vertically opposite $\angle$s)
$\therefore \angle CHG = \angle JIF$ (both equal to α)

9 Let $\angle ABP = \alpha$
$\angle PBD = \alpha$ (PQ bisects $\angle ABD$)
$\angle ABD = 2\alpha$ (sum of adjacent $\angle$s)
$\angle BDE = 2\alpha$ (alternate $\angle$s, $AB \parallel CE$)
$\angle BDS = \alpha$ (RS bisects $\angle BDE$)
$\therefore \angle PBD = \angle BDS$ (both equal to α)
$\therefore PQ \parallel RS$ (alternate $\angle$s are equal)

Exercise 4.5

1 Construct DE, through B, parallel to AC
$\angle DBA = \angle BAC$ (alternate $\angle$s, $DE \parallel AC$)
$\angle EBC = \angle BCA$ (alternate $\angle$s, $DE \parallel AC$)
$\angle DBA + \angle ABC + \angle EBC = 180°$ ($\angle$s on a straight line)
$\therefore \angle BAC + \angle ABC + \angle BCA = 180°$

2 Construct CE, parallel to AB
$\angle ECD = \angle BAC$ (corresponding $\angle$s, $AB \parallel CE$)
$\angle BCE = \angle ABC$ (alternate $\angle$s, $AB \parallel CE$)
$\angle BCD = \angle ECD + \angle BCE$ (sum of adjacent $\angle$s)
$\therefore \angle BCD = \angle BAC + \angle ABC$

3 $\angle C = \angle A + \angle B$ (given)
$\angle A + \angle B + \angle C = 180°$ ($\angle$ sum of a Δ)
$\therefore \angle A + \angle B + (\angle A + \angle B) = 180°$
$2(\angle A + \angle B) = 180°$
$\therefore \angle A + \angle B = 90°$
$\therefore \angle C = 90°$
$\therefore \Delta ABC$ is right-angled

4 Let $\angle BCA = \alpha$
$\angle DAC = 90° - \alpha$ ($\angle$ sum of ΔACD)
$\angle EBC = 90° - \alpha$ ($\angle$ sum of ΔBCE)
$\therefore \angle DAC = \angle EBC$ (both equal to $90° - \alpha$)

5 Let $\angle BAC = \alpha$
$\angle BCA = \alpha$ (base $\angle$s of isosceles Δ, $AB = BC$)
$\angle DCE = \alpha$ (vertically opposite $\angle$s)
$\angle DEC = \alpha$ (alternate $\angle$s, $AB \parallel DE$)
$\therefore \Delta CDE$ is isosceles ($\angle DCE = \angle DEC$)

6 Let $\angle CAB = \alpha$
$\angle ACB = \alpha$ (base $\angle$s of isosceles Δ, $AB = BC$)
$\angle EBD = \alpha$ (corresponding $\angle$s, $AC \parallel BE$)
$\angle CBE = \alpha$ (alternate $\angle$s, $AC \parallel BE$)
$\therefore \angle CBE = \angle EBD$ (both equal to α)
$\therefore EB$ bisects $\angle CBD$

7 Let $\angle PRS = \alpha$
$\angle QRS = \alpha$ (SR bisects $\angle PRQ$)
$\angle PRQ = 2\alpha$ (sum of adjacent $\angle$s)
$\angle PQR = 2\alpha$ (base $\angle$s of isosceles Δ, $PQ = PR$)
$\angle PSR = 3\alpha$ (exterior $\angle$ of ΔQRS)
$\therefore \angle PSR = 3\angle PRS$

8 $\angle BAD = 30°$ (given)
$\angle ABD = 30°$ (base $\angle$s of isosceles Δ, $AD = BD$)
$\angle DBC = 60°$ (adjacent $\angle$s in a right angle)
$\angle BCD = 60°$ ($\angle$ sum of ΔABC)
$\angle BDC = 60°$ (exterior $\angle$ of ΔABD)
$\therefore \Delta BCD$ is equilateral (all $\angle$s are $60°$)

9 Let $\angle YXW = \alpha$ and $\angle XYW = \beta$

a $\angle XWZ = \alpha + \beta$ (exterior $\angle$ of ΔXYW)

b $\angle WXZ = \alpha$ (XW bisects $\angle YXZ$)

$\angle XZY = \beta$ (base $\angle$s of isosceles Δ, $XY = XZ$)

$\angle XWY = \alpha + \beta$ (exterior $\angle$ of ΔXWZ)

c $\angle XWY + \angle XWZ = 180°$ (adjacent $\angle$s on a straight line)

$(\alpha + \beta) + (\alpha + \beta) = 180°$

$2\alpha + 2\beta = 180°$

$\therefore \alpha + \beta = 90°$

$\therefore \angle XWZ = \angle XWY = 90°$

$\therefore XW \perp YZ$

11 **a** $OA = OB = OC$ (equal radii)

b Let $\angle OAC = \alpha$ and $\angle OBC = \beta$

$\angle OCA = \alpha$ (base $\angle$s of isosceles Δ, $OA = OC$)

$\angle OCB = \beta$ (base $\angle$s of isosceles Δ, $OB = OC$)

$\therefore 2\alpha + 2\beta = 180°$ ($\angle$ sum of ΔABC)

$\therefore \alpha + \beta = 90°$

$\angle ACB = \alpha + \beta$ (sum of adjacent $\angle$s)

$\therefore \angle ACB = 90°$

13 Let $\angle ABG = \alpha$ and $\angle CDG = \beta$

$\angle GBD = \alpha$ (EB bisects $\angle ABD$)

$\angle GDB = \beta$ (FD bisects $\angle BDC$)

$\therefore 2\alpha + 2\beta = 180°$ (co-interior $\angle$s, $AB \parallel CD$)

$\therefore \alpha + \beta = 90°$

$\angle BGD = 180° - (\alpha + \beta)$ ($\angle$ sum of a Δ)

$= 180° - 90°$

$= 90°$

$\therefore EB \perp FD$

15 **a** $\angle BDC = \alpha + \beta$ (exterior $\angle$ of ΔABD)

b $\angle BCE = \angle BAD$ (given)

$= \alpha$

$\angle DBC = \beta$ (BD bisects $\angle ABC$)

$\angle DEC = \alpha + \beta$ (exterior $\angle$ of ΔBCE)

$\therefore \angle BDC = \angle DEC$ (both equal to $\alpha + \beta$)

$\therefore CD = CE$ (equal sides lie opposite equal $\angle$s)

17 Let $\angle ABF = \alpha$ and $\angle ACE = \beta$

$\angle EBC = \alpha$ (EB bisects $\angle ABC$)

$\angle ACB = 2\alpha$ (base $\angle$s of isosceles Δ, $AB = AC$)

$\angle BEC = \alpha$ (alternate $\angle$s, $AB \parallel EC$)

$\angle ECD = \beta$ (EC bisects $\angle ACD$)

$2\alpha + 2\beta = 180°$ (adjacent $\angle$s on a straight line)

$\therefore \alpha + \beta = 90°$

$\angle BFC = \alpha + \beta$ (exterior $\angle$ of ΔCEF)

$\therefore \angle BFC = 90°$

$\therefore AC \perp BE$

10 **a** Let $\angle BAD = \alpha$

$\angle ABD = \alpha$ (base $\angle$s of isosceles Δ, $AD = DB$)

$\angle DBC = 90° - \alpha$ (adjacent $\angle$s in a right angle)

$\angle ACB = 90° - \alpha$ ($\angle$ sum of ΔABC)

$\therefore \angle DBC = \angle ACB$ (both equal to $90° - \alpha$)

$\therefore \Delta BCD$ is isosceles

b $AD = DB$ (given)

$DB = DC$ (equal sides lie opposite equal $\angle$s, $\angle DBC = \angle ACB$)

$\therefore AD = DC$ (both equal to DB)

$\therefore D$ is the midpoint of AC

12 **a** Let $\angle ABE = \alpha$ and $\angle BAC = \beta$

$\angle BEC = \alpha + \beta$ (exterior $\angle$ of ΔABE)

$\angle EBC = \alpha$ (BE bisects $\angle ABC$)

$\angle BCD = (\alpha + \beta) + \alpha = 2\alpha + \beta$ (exterior $\angle$ of ΔBCD)

b $\angle BAC + \angle BCD = \beta + 2\alpha + \beta$

$= 2(\alpha + \beta)$

$= 2\angle BEC$

14 Let $\angle UWY = \alpha$

$\angle WUX = 90° - \alpha$ ($\angle$ sum of ΔUWX)

$\angle VYW = \alpha$ (base $\angle$s of isosceles Δ, $VW = VY$)

$\angle XZY = 90° - \alpha$ ($\angle$ sum of ΔXYZ)

$\angle UZV = 90° - \alpha$ (vertically opposite $\angle$s)

$\therefore \angle WUX = \angle UZV$ (both equal to $90° - \alpha$)

$\therefore \Delta UVZ$ is isosceles

16 Let $\angle ABG = \alpha$ and $\angle CDE = \beta$

$\angle GBC = \alpha$ (GB bisects $\angle ABC$)

$\angle ABC = 2\alpha$ (sum of adjacent $\angle$s)

$\angle BCF = 180° - 2\alpha$ (co-interior $\angle$s, $AB \parallel FE$)

$\angle CED = \beta$ (base $\angle$s of isosceles Δ, $CD = CE$)

$\angle ECD = 180° - 2\beta$ ($\angle$ sum of ΔCDE)

$\angle BCF = \angle ECD$ (vertically opposite $\angle$s)

$\therefore 180° - 2\alpha = 180° - 2\beta$

$\therefore \alpha = \beta$

$\therefore \angle GBC = \angle CDE$

$\therefore GB \parallel DE$ (alternate $\angle$s are equal)

ANSWERS

Exercise 4.6

1 **a** AAS, **b** RHS, **c** SSS, **d** SAS **2** **a** yes, SAS, **b** no, **c** yes, RHS, **d** no **3** no, AAA is not a congruence test

4 **a** $QR = SR$ (given)
$PR = TR$ (given)
$\angle QRP = \angle SRT$ (vertically opposite $\angle$s)
$\therefore \Delta PQR \equiv \Delta TSR$ (SAS)

b $DF = FH$ (given)
$\angle DEF = \angle FGH$ (alternate $\angle$s, $DE \parallel GH$)
$\angle DFE = \angle GFH$ (vertically opposite $\angle$s)
$\therefore \Delta DEF \equiv \Delta HGF$ (AAS)

c $XY = XW$ (given)
$YZ = WZ$ (given)
XZ is a common side
$\therefore \Delta XYZ \equiv \Delta XWZ$ (SSS)

d $LM = MN$ (given)
MK is a common side
$\angle MKL = \angle MKN = 90°$ ($MK \perp LN$)
$\therefore \Delta MLK \equiv \Delta MNK$ (RHS)

5 **a** $AE = BE$ (CD bisects AB)
$CE = DE$ (AB bisects CD)
$\angle AEC = \angle BED$ (vertically opposite $\angle$s)
$\therefore \Delta ACE \equiv \Delta BDE$ (SAS)
$\therefore AC = BD$ (matching sides of congruent Δs)

b $OI = OK$ (equal radii)
OJ is a common side
$\angle OJI = \angle OJK = 90°$ ($OJ \perp IK$)
$\therefore \Delta OIJ \equiv \Delta OKJ$ (RHS)
$\therefore \angle IOJ = \angle KOJ$ (matching $\angle$s of congruent Δs)
$\therefore OJ$ bisects $\angle IOK$

c $\angle WXY = \angle WZY$ (given)
$\angle XYW = \angle ZYW$ (WY bisects $\angle XYZ$)
WY is a common side
$\therefore \Delta XYW \equiv \Delta ZYW$ (AAS)
$\therefore WX = WZ$ (matching sides of congruent Δs)
$\therefore \Delta XWZ$ is isosceles

d $OA = OD$ (equal radii)
$OB = OC$ (equal radii)
$AB = CD$ (given)
$\therefore \Delta AOB \equiv \Delta DOC$ (SSS)
$\therefore \angle AOB = \angle COD$ (matching $\angle$s of congruent Δs)

e $\angle QPR = \angle SRT$ (corresponding $\angle$s, $PQ \parallel RS$)
$\angle QRP = \angle STR$ (corresponding $\angle$s, $QR \parallel ST$)
$PR = RT$ (QR bisects PT)
$\therefore \Delta PQR \equiv \Delta RST$ (AAS)
$\therefore PQ = RS$ (matching sides of congruent Δs)

f $\angle IJL = \angle JLK$ (alternate $\angle$s, $IJ \parallel LK$)
$\angle ILJ = \angle LJK$ (alternate $\angle$s, $LI \parallel KJ$)
LJ is a common side
$\therefore \Delta IJL \equiv \Delta KLJ$ (AAS)
$\therefore \angle IJL = \angle KJL$ (matching $\angle$s of congruent Δs)
$\therefore LJ$ bisects $\angle IJK$

g $CD = DE$ (given)
DF is a common side
$\angle DFC = \angle DFE = 90°$ ($DF \perp CE$)
$\therefore \Delta CDF \equiv \Delta EDF$ (RHS)
$\therefore CF = FE$ (matching sides of congruent Δs)
$\therefore DF$ bisects CE

h $RS = QT$ (given)
RT is a common side
$\angle SRT = \angle RTQ$ (alternate $\angle$s, $RS \parallel QT$)
$\therefore \Delta RST \equiv \Delta TQR$ (SAS)
$\therefore \angle RTS = \angle QRT$ (matching $\angle$s of congruent Δs)
$\therefore QR \parallel TS$ (alternate $\angle$s are equal)

i $AB = BD$ (BC bisects AD)
$BC = DE$ (given)
$\angle ABC = \angle BDE$ (corresponding $\angle$s, $BC \parallel DE$)
$\therefore \Delta ABC \equiv \Delta BDE$ (SAS)
$\therefore \angle BAC = \angle DBE$ (matching $\angle$s of congruent Δs)
$\therefore AC \parallel BE$ (corresponding $\angle$s are equal)

j $OX = OY$ (equal radii)
$XM = MY$ (OM bisects XY)
OM is a common side
$\therefore \Delta OMX \equiv \Delta OMY$ (SSS)
$\therefore \angle OMX = \angle OMY$ (matching $\angle$s of congruent Δs)
But, $\angle OMX + \angle OMY = 180°$ (adjacent $\angle$s on a straight line)
$\therefore \angle OMX = \angle OMY = 90°$
$\therefore OM \perp XY$

6 **a** $PQ = PR$ (given)
$QX = RY$ (given)
$\angle PQX = \angle PRY$ (base $\angle$s of isosceles Δ, $PQ = PR$)
$\therefore \Delta PQX \equiv \Delta PRY$ (SAS)

b $PX = PY$ (matching sides of congruent Δs)
$\therefore \Delta$PXY is isosceles

7 **a** $AB = AC$ (given)
$PB = QC$ (P, Q are midpoints of equal sides)

b $PB = QC$ (proven above)
BC is a common side
$\angle ABC = \angle ACB$ (base $\angle$s of isosceles Δ, $AB = AC$)
$\therefore \Delta PBC \equiv \Delta QCB$ (SAS)
$\therefore PC = QB$ (matching sides of congruent Δs)

8 **a** $\angle ABC = \angle ACB$ (base $\angle$s of isosceles Δ, $AB = AC$)
$\angle BLC = \angle CMB = 90°$ ($CL \perp AB$, $BM \perp AC$)
BC is a common side
$\therefore \Delta BLC \equiv \Delta CMB$ (AAS)

b $BL = CM$ (matching sides of congruent Δs)
$\angle BLN = \angle CMN = 90°$ ($CL \perp AB$, $BM \perp AC$)
$\angle BNL = \angle CNM$ (vertically opposite $\angle$s)
$\therefore \Delta BLN \equiv \Delta CMN$ (AAS)

c $LN = MN$ (matching sides of congruent Δs)

9 a Construct AD, the angle bisector of $\angle A$
$AB = AC$ (given)
$\angle BAD = \angle CAD$ (AD bisects $\angle BAC$)
AD is a common side
$\therefore \Delta BAD \equiv \Delta CAD$ (SAS)
$\therefore \angle ABC = \angle ACB$ (matching $\angle$s of congruent Δs)

b Construct AD, the angle bisector of $\angle A$
$\angle ABC = \angle ACB$ (given)
$\angle BAD = \angle CAD$ (AD bisects $\angle BAC$)
AD is a common side
$\therefore \Delta ABD \equiv \Delta ACD$ (AAS)
$\therefore AB = AC$ (matching sides of congruent Δs)

10 a $AC = CB$ (sides of an equilateral Δ)
$\angle ACD = \angle BCD$ (CD bisects $\angle ACB$)
CD is a common side
$\therefore \Delta ACD \equiv \Delta BCD$ (SAS)
$\therefore \angle A = \angle B$ (matching $\angle$s of congruent Δs)

b $AB = AC$ (sides of an equilateral Δ)
$\angle BAE = \angle CAE$ (AE bisects $\angle BAC$)
AE is a common side
$\therefore \Delta ABE \equiv \Delta ACE$ (SAS)
$\therefore \angle B = \angle C$ (matching $\angle$s of congruent Δs)

c $\angle A = B$ (proven above)
$\angle B = \angle C$ (proven above)
$\therefore \angle A = \angle B = \angle C$
But, $\angle A + \angle B + \angle C = 180°$ ($\angle$ sum of a Δ)
$\therefore \angle A = \angle B = \angle C = 60°$

11 i Let $\angle ABC = \alpha$ and $\angle PBC = \beta$
$\angle ABP = \alpha - \beta$ (by subtraction)
$\angle ACB = \alpha$ (base $\angle$s of isosceles Δ, $AB = AC$)
$\angle PCB = \beta$ (base $\angle$s of isosceles Δ, $PB = PC$)
$\angle ACP = \alpha - \beta$ (by subtraction)
$\therefore \angle ABP = \angle ACP$ (both equal to $\alpha - \beta$)

ii $AB = AC$ (given)
$\angle ABP = \angle ACP$ (proven above)
$PB = PC$ (given)
$\therefore \Delta ABP \equiv \Delta ACP$ (SAS)
$\angle BAP = \angle CAP$ (matching $\angle$s of congruent Δs)
$\therefore PA$ bisects $\angle ABC$

12 $QM = PN$ (given)
$PM = SN$ (given)
$\angle LMN = \angle LNM$ (base $\angle$s of isosceles Δ, $LM = LN$)
$\therefore \Delta QMP \equiv \Delta PNS$ (SAS)
$\therefore PQ = PS$ (matching sides of congruent Δs)
$\therefore \angle PQS = \angle PSQ$ (base $\angle$s of isosceles Δ, $PQ = PS$)

13 i $\angle BAC = 60°$ ($\angle$ in an equilateral Δ)
$\angle DAC = 120°$ (adjacent $\angle$s on a straight line)
$\angle ACB = 60°$ ($\angle$ in an equilateral Δ)
$\angle BCE = 120°$ (adjacent $\angle$s on a straight line)
$\angle ACD = 30°$ (adjacent $\angle$s in a right angle)
$\angle ABC = 60°$ ($\angle$ in an equilateral Δ)
$\angle CBE = 30°$ (adjacent $\angle$s in a right angle)

ii $AC = BC$ (sides of an equilateral Δ)
$\angle ACD = \angle CBE = 30°$ (proven above)
$\angle DAC = \angle BCE = 120°$ (proven above)
$\therefore \Delta ACD \equiv \Delta CBE$ (AAS)
$\therefore CE = AD$ (matching sides of congruent Δs)

Exercise 4.7

1 $\angle BAC + \angle ABC + \angle BCA = 180°$ ($\angle$ sum of ΔABC)
$\angle DAC + \angle ADC + \angle DCA = 180°$ ($\angle$ sum of ΔADC)
$\therefore (\angle BAC + \angle DAC) + \angle ABC + (\angle BCA + \angle DCA) + \angle ADC = 360°$ (by addition)
$\therefore \angle A + \angle B + \angle C + \angle D = 360°$

2 a $\angle BAC = \angle ACD$ (alternate $\angle$s, $AB \parallel DC$)
$\angle BCA = \angle CAD$ (alternate $\angle$s, $BC \parallel AD$)
AC is a common side
$\therefore \Delta ABC \equiv \Delta CDA$ (AAS)

b $AB = DC$ (matching sides of congruent Δs)
$AD = BC$ (matching sides of congruent Δs)

c $\angle ABC = \angle ADC$ (matching $\angle$s of congruent Δs)

d the opposite sides of a parallelogram are equal, and the opposite angles of a parallelogram are equal.

3 a $\angle ABP = \angle PDC$ (alternate $\angle$s, $AB \parallel DC$)
$\angle BAP = \angle PCD$ (alternate $\angle$s, $AB \parallel DC$)
$AB = DC$ (opp. sides of a parallelogram)
$\therefore \Delta APB \equiv \Delta CPD$ (AAS)

b $AP = PC$ (matching sides of congruent Δs)
$BP = PD$ (matching sides of congruent Δs)

c the diagonals of a parallelogram bisect each other.

4 a $\angle BCA = \alpha$ (base $\angle$s of isosceles Δ, $AB = BC$)
$\angle DAC = \alpha$ (alternate $\angle$s, $AD \parallel BC$)

b $\angle BDC = \beta$ (alternate $\angle$s, $AB \parallel DC$)
$BC = CD$ (sides of a rhombus are equal)
$\angle CBD = \beta$ (base $\angle$s of isosceles Δ, $BC = CD$)

c the diagonals of a rhombus bisect the angles at the vertices.

ANSWERS

5 a $\angle BCA = \alpha$ (base $\angle$s of isosceles Δ, $AB = BC$)
$\angle CBD = \beta$ (diagonal of a rhombus bisects the $\angle$ at the vertex)
b $2\alpha + 2\beta = 180°$ ($\angle$ sum of ΔABC)
$\therefore \alpha + \beta = 90°$
c $\angle BPC = \alpha + \beta$ (exterior $\angle$ of ΔABP)
$\therefore \angle BPC = 90°$
$\therefore AC \perp BD$
d the diagonals of a rhombus are perpendicular.

7 a i $2\alpha + 2\beta = 360°$ ($\angle$ sum of a quadrilateral)
$\therefore \alpha + \beta = 180°$
ii $\angle BAD + \angle ADC = \alpha + \beta = 180°$
$\therefore AB \parallel DC$ (co-interior $\angle$s are supplementary)
$\angle BAD + \angle ABC = \alpha + \beta = 180°$
$\therefore AD \parallel BC$ (co-interior $\angle$s are supplementary)
b i $AB = CD$ (given)
$BC = AD$ (given)
AC is a common side
$\therefore \Delta ABC \equiv \Delta CDA$ (SSS)
ii $\angle BAC = \angle ACD$ (matching $\angle$s of congruent Δs)
$\therefore AB \parallel CD$ (alternate $\angle$s are equal)
$\angle BCA = \angle CAD$ (matching $\angle$s of congruent Δs)
$\therefore BC \parallel AD$ (alternate $\angle$s are equal)
c i $AB = DC$ (given)
$\angle BAC = \angle ACD$ (alternate $\angle$s, $AB \parallel DC$)
AC is a common side
$\therefore \Delta ABC \equiv \Delta CDA$ (SAS)
ii $\angle BCA = \angle CAD$ (matching $\angle$s of congruent Δs)
$\therefore AD \parallel BC$ (alternate $\angle$s are equal)
d i $AP = PC$ (BD bisects AC)
$BP = PD$ (AC bisects BD)
$\angle APB = \angle CPD$ (vertically opposite $\angle$s)
$\therefore \Delta APB \equiv \Delta CPD$ (SAS)
ii $AB = DC$ (matching sides of congruent Δs)
$\angle ABP = \angle PDC$ (matching $\angle$s of congruent Δs)
$\therefore AB \parallel DC$ (alternate $\angle$s are equal)

10 a $\angle ADP = \alpha$ (base $\angle$s of isosceles Δ, $PA = PD$)
$\angle ABP = \beta$ (base $\angle$s of isosceles Δ, $PA = PB$)
b $2\alpha + 2\beta = 180°$ ($\angle$ sum of ΔABD)
$\therefore \alpha + \beta = 90°$
$\angle DAB = \alpha + \beta$ (sum of adjacent $\angle$s)
$\therefore \angle DAB = 90°$
c $ABCD$ is a parallelogram (diagonals bisect each other)
But, $\angle DAB = 90°$ (proven above)
$\therefore ABCD$ is a rectangle (parallelogram with one angle a right angle).

6 a $AB = CD$ (opposite sides of a rectangle are equal)
$\angle ABC = \angle BCD = 90°$ (angles in a rectangle)
BC is a common side
$\therefore \Delta ABC \equiv \Delta DCB$ (SAS)
b $AC = BD$ (matching sides of congruent Δs)
c the diagonals of a rectangle are equal.

8 a $AP = PC$ (BD bisects AC)
$\angle BPA = \angle BPC = 90°$ ($BD \perp AC$)
BP is a common side
$\therefore \Delta ABP \equiv \Delta CBP$ (SAS)
b $AB = BC$ (matching sides of congruent Δs)
c $ABCD$ is a parallelogram (diagonals bisect each other)
$AB = BC$ (proven above)
$\therefore ABCD$ is a rhombus (parallelogram with a pair of adjacent sides equal)

9 i $4\alpha = 360°$ ($\angle$ sum of a quadrilateral)
$\therefore \alpha = 90°$
ii $\angle BAD + \angle ADC = \alpha + \alpha = 180°$
$\therefore AB \parallel DC$ (co-interior $\angle$s are supplementary)
$\angle BAD + \angle ABC = \alpha + \alpha = 180°$
$\therefore AD \parallel BC$ (co-interior $\angle$s are supplementary)
$\therefore ABCD$ is a parallelogram (two pairs of opposite sides parallel)
But, $\angle ABC = 90°$
$\therefore ABCD$ is a rectangle (a parallelogram with one angle a right angle).

11 a $\angle PBR = \angle RDC$ (alternate $\angle$s, $AB \parallel DC$)
$\angle BPR = \angle RQD$ (alternate $\angle$s, $AB \parallel DC$)
$BR = RD$ (AC bisects BD)
$\therefore \Delta BPR \equiv \Delta DQR$ (AAS)
b $PB = DQ$ (matching sides of congruent Δs)
$AB = DC$ (opposite sides of a parallelogram are equal)
$\therefore AB - PB = DC - DQ$
$\therefore AP = QC$

12 a $\angle BDC = \alpha$ (alternate $\angle$s, $AB \parallel DC$)
$\angle ADB = \alpha$ (BD bisects $\angle ADC$)
$\therefore AB = AD$ (equal sides lie opposite equal $\angle$s)
b $ABCD$ is a rhombus (parallelogram with a pair of adjacent sides equal).

13 a Let $\angle DBC = \alpha$
$\angle ADB = \alpha$ (alternate $\angle$s, $AB \parallel DC$)
$\angle FBC = 180° - \alpha$ (adjacent $\angle$s on a st. line)
$\angle ADE = 180° - \alpha$ (adjacent $\angle$s on a st. line)
$\therefore \angle FBC = \angle ADE$ (both equal to $180° - \alpha$)
b $\angle FBC = \angle ADE$ (proven above)
$BC = AD$ (opposite sides of a parallelogram are equal)
$BF = ED$ (given)
$\therefore \Delta FBC \equiv \Delta EDA$ (SAS)
c $\angle BFC = \angle DEA$ (matching $\angle$s of congruent Δs)
$\therefore FC \parallel AE$ (alternate $\angle$s are equal)
$FC = AE$ (matching sides of congruent Δs)
$\therefore AFCE$ is a parallelogram (one pair of opposite sides equal and parallel).

Exercise 4.8

1 a $x = 39$, **b** $a = 25$, **c** $t = 2\sqrt{5}$, **d** $m = 15$, **e** $u = 4\sqrt{2}$, **f** $e = 3\sqrt{5}$
2 a no, **b** yes, **c** no, **d** yes **3 a** $BD = 12$ cm, $BC = 15$ cm, **b** $AC^2 = AB^2 + BC^2$; $\therefore \Delta ABC$ is right-angled.
4 12 cm **5** 4 m **6** 3 cm **7** $x = 25$ **8** 5 cm, 12 cm, 13 cm **11** 136 cm
14 b $AC^2 = x^2 + xy$, $BC^2 = y^2 + xy$, **c** $AB^2 = AC^2 + BC^2$ **15 a** $AB^2 = AD^2 + BD^2$, **b** $CD^2 = BC^2 - BD^2$
16 d In a quadrilateral in which the diagonals are perpendicular, the sum of the squares on the opposite sides are equal.

Chapter 4 Review

1 a $x = 50$, **b** $p = 74$, **c** $a = 62$, **d** $c = 36$, **e** $e = 114$, **f** $s = 55$, **g** $m = 153$, **h** $b = 86$
2 a $\angle ABD + \angle DBC = 90°$, $\therefore AB \perp BC$, **b** $\angle PQS + \angle SQR = 180°$, $\therefore P, Q, R$ are collinear.
3 a no, corresponding $\angle$s are not equal, **b** yes, co-interior $\angle$s are supplementary, **c** yes, alternate $\angle$s are equal
4 a 7, **b** 11 **5 a** 720°, 60°, **b** 1440°, 144°, **c** 540°, 108°, **d** 1260°, 140°, **e** 1080°, 135°, **f** 1800°, 150°
6 a 72°, **b** 45°, **c** 60° **7 a** 10, **b** 24, **c** 18 **8** no

9 a Let $\angle ABG = \alpha$
$\angle CDG = \angle ABG$ (corresponding $\angle$s, $AB \parallel CD$)
$= \alpha$
$\angle DEF = \angle ABG$ (given)
$= \alpha$
$\angle CDE = \angle DEF$ (alternate $\angle$s, $CD \parallel EF$)
$= \alpha$
Now, $\angle CDE = \angle CDG$ (both equal to α)
$\therefore CD$ bisects $\angle GDE$

b Let $\angle QSR = \alpha$
$\angle PQS = 180° - \alpha$ (co-interior $\angle$s, $PQ \parallel RS$)
$\angle QST = 90° - \alpha$ ($TS \perp RS$)
$\angle TSU = 90° - (90° - \alpha)$ ($QS \perp SU$)
$= \alpha$
Now, $\angle PQS + \angle TSU = 180°$, $\therefore \angle PQS$ and $\angle TSU$ are supplementary.

c Let $\angle IMH = \alpha$
$\angle IMN = 90° + \alpha$ (by addition)
$\angle JIM = 180° - (90° + \alpha)$ (co-interior $\angle$s, $IJ \parallel MN$)
$= 90° - \alpha$
$\angle JIK = 2\angle JIM$ (IM bisects $\angle JIK$)
$= 180° - 2\alpha$
$\angle IKL = 2\alpha$ (co-interior $\angle$s, $IJ \parallel KL$)
$\therefore \angle IKL = 2\angle IMH$

d Let $\angle PQS = \alpha$
$\angle SPQ = \angle PQS$ (base $\angle$s of isosceles Δ, $PS = SQ$)
$= \alpha$
$\angle SQR = 90° - \alpha$ ($PQ \perp QR$)
$\angle PRQ = 90° - \alpha$ ($\angle$ sum of ΔPQR)
Now, $\angle SQR = \angle PRQ$ (both equal to $90° - \alpha$)
$\therefore \Delta QRS$ is isosceles

e Let $\angle GBD = \alpha$
$\angle ABG = \angle GBD$ (BG bisects $\angle ABD$)
$= \alpha$
$\angle BDF = 90° - \alpha$ ($\angle$ sum of ΔBED)
$\angle CDF = \angle BDF$ (FD bisects $\angle BDC$)
$= 90° - \alpha$
Now, $\angle ABD + \angle BDC = 2\alpha + 2(90° - \alpha)$
$= 180°$
$\therefore AB \parallel CD$ (co-interior $\angle$s are supplementary)

f Let $\angle EAC = \alpha$ and $\angle ECA = \beta$
$\angle ABC = \angle EAC$ (given)
$= \alpha$
$\angle BCD = \angle ECA$ (DC bisects $\angle ACB$)
$= \beta$
$\angle AED = \alpha + \beta$ (ext. $\angle$ of ΔACE)
$\angle ADE = \alpha + \beta$ (ext. $\angle$ of ΔBCD)
Now, $\angle AED = \angle ADE$ (both equal to $\alpha + \beta$)
$\therefore \Delta ADE$ is isosceles

ANSWERS

10 a i $AB = BC$ (given)
$OA = OC$ (equal radii)
OB is a common side
$\therefore \Delta OAB \equiv \Delta OCB$ (SSS)

ii $\angle AOB = \angle COB$ (matching $\angle$s of congruent Δs)
$\therefore OB$ bisects $\angle AOC$

b i $\angle PQS = \angle QSR = 90°$ (alternate $\angle$s, $PQ \parallel SR$)
$PS = QR$ (given)
QS is a common side
$\therefore \Delta PQS \equiv \Delta RSQ$ (RHS)

ii $\angle PSQ = \angle RQS$ (matching $\angle$s of congruent Δs)
$\therefore PS \parallel QR$ (alternate $\angle$s are equal)
$PQ \parallel SR$ (given)
$\therefore PQRS$ is a parallelogram (opposite sides are parallel)

c i $\angle XWZ = \angle XYZ$ (given)
$\angle XZW = \angle XZY$ (XZ bisects $\angle WZY$)
XZ is a common side
$\therefore \Delta XWZ \equiv \Delta XYZ$ (AAS)

ii $XW = XY$ (matching sides of congruent Δs)
$\therefore \Delta WXY$ is isosceles

d i $CD = EF$ (given)
$\angle DCE = \angle FEG$ (corresponding $\angle$s, $CD \parallel EF$)
$CE = EG$ (EF bisects CG)
$\therefore \Delta CDE \equiv \Delta EFG$ (SAS)

ii $DE = FG$ (matching sides of congruent Δs)

11 a $PQ = SR$ (opposite sides of a parallelogram)
$\angle PQT = \angle RST$ (alternate $\angle$s, $PQ \parallel SR$)
$\angle PTQ = \angle STR$ (vert. opp. $\angle$s)
$\therefore \Delta PQT \equiv \Delta RST$ (AAS)

b $PT = TR$ (matching sides of congruent Δs)
$QT = TS$ (matching sides of congruent Δs)
$\therefore$ the diagonals bisect each other

12 a $KN = LM$ (opposite sides of a rectangle)
$\angle KNM = \angle LMN = 90°$ ($\angle$s in a rectangle)
NM is a common side
$\therefore \Delta KNM \equiv \Delta LMN$ (SAS)

b $KM = LN$ (matching sides of congruent Δs)
$\therefore$ diagonals of a rectangle are equal

13 a $XY = YZ$ (sides of a rhombus)
$\angle XZY = \angle ZXY$ (base $\angle$s of isosceles Δ, $XY = YZ$)
$= \alpha$
$\angle WXZ = \angle XZY$ (alternate $\angle$s, $WX \parallel ZY$)
$= \alpha$
Now, $\angle WXZ = \angle ZXY$ (both equal to α)
$\therefore ZX$ bisects $\angle WXY$

b $WX = XY$ (sides of a rhombus)
$\angle XWY = \angle XYW$ (base $\angle$s of isosceles Δ, $WX = XY$)
$= \beta$
$2\alpha + 2\beta = 180°$ ($\angle$ sum of ΔWXY)
$\therefore \alpha + \beta = 90°$
$\angle WAX = \alpha + \beta$ (exterior $\angle$ of ΔXAY)
$= 90°$
$\therefore XZ \perp WY$

14 a diagonals bisect each other

b SW is a common side
$\angle SWV = \angle SWT = 90°$ ($SU \perp VT$)
$VW = WT$ (SU bisects VT)
$\therefore \Delta SVW \equiv \Delta STW$ (SAS)

c $SV = ST$ (matching sides of congruent Δs)
$\therefore STUV$ is a rhombus (parallelogram with a pair of adjacent sides equal)

15 a $AB = AC$ (given)
$BR = CS$ (R, S are midpoints of AB, AC)

b $BR = CS$ (proven above)
$\angle ABC = \angle ACB$ (base $\angle$s of isosceles Δ, $AB = AC$)
BC is a common side
$\therefore \Delta RBC \equiv \Delta SCB$ (SAS)

c $CR = BS$ (matching sides of congruent Δs)

d $TR = TS$ (given)
$\therefore CR - TR = BS - TS$
$\therefore TB = TC$
$\therefore \Delta BTC$ is isosceles

16 $AC^2 = AB^2 + BC^2$
$= AB^2 + (2AB)^2$
$= 5AB^2$

$AD^2 = AC^2 + CD^2$
$= 5AB^2 + (2AB)^2$
$= 9AB^2$
$\therefore AD = 3AB$

17 a $\frac{MP}{LP} = \frac{LP}{PN}$ (given)

$\therefore \frac{a}{LP} = \frac{LP}{b}$

$LP^2 = ab$

$\therefore LP = \sqrt{ab}$

b $LM^2 = a^2 + ab$, $LN^2 = b^2 + ab$

c $LM^2 + LN^2 = a^2 + ab + b^2 + ab$
$= a^2 + 2ab + b^2$
$= (a + b)^2$
$= MN^2$
$\therefore \Delta LMN$ is right-angled (converse of Pythagoras' theorem)

5 Factorisation and algebraic fractions

Exercise 5.1

1 $x^2+8x+15$ **2 a** $xy+3x+2y+6$, **b** $pq-2p+5q-10$, **c** $uv-6u-4v+24$ **3 a** x^2+6x+8, **b** m^2+7m+6, **c** $u^2+3u-40$, **d** $b^2+7b-18$, **e** $a^2-5a-24$, **f** t^2-3t+2, **g** $c^2-10c+24$, **h** $z^2-13z+42$, **i** $d^2+7d-60$, **j** $2x^2+7x+3$, **k** $3m^2+10m+8$, **l** $5a^2-18a-8$, **m** $2g^2-11g+15$, **n** $3t^2-17t-6$, **o** $4n^2+13n-35$, **p** $10r^2+41r+21$, **q** $6k^2+17k-45$, **r** $21v^2-62v+16$ **5 a** x^2+3x+2, **b** p^2+5p+6, **c** $a^2+10a+21$, **d** $t^2-9t+20$, **e** $b^2-8b+12$, **f** $c^2-7c+12$, **g** z^2+3z-4, **h** $d^2-3d-18$, **i** $s^2+8s-20$, **j** $e^2+5e-14$, **k** $u^2+6u-27$, **l** $k^2-7k-44$, **m** $f^2+12f+35$, **n** w^2-w-30, **o** $r^2-12r+32$, **p** $g^2+2g-48$, **q** $h^2+6h-55$, **r** $v^2+11v+24$, **s** $q^2-13q+36$, **t** $m^2+3m-54$, **u** $i^2-9i-36$, **v** $l^2+4l-60$, **w** $y^2-10y-39$, **x** $j^2-15j+56$ **6** $6a^2-9a-60$ **7 a** $a^2+2ab+b^2$, **b** $m^2+2mn+n^2$, **c** $g^2-2gh+h^2$, **d** $p^2-2pq+q^2$ **8 a** a^2+6a+9, **b** $p^2-8p+16$, **c** $c^2+12c+36$, **d** t^2-2t+1, **e** $u^2-10u+25$, **f** $k^2+20k+100$, **g** $s^2+14s+49$, **h** $p^2-18p+81$ **9 a** $4x^2+12x+9$, **b** $9t^2-30t+25$, **c** $16m^2+8m+1$, **d** $25c^2-20c+4$, **e** $49-28g+4g^2$, **f** $25+60r+36r^2$, **g** $9p^2-6pq+q^2$, **h** $9a^2+24ab+16b^2$, **i** $4e^2-44ef+121f^2$, **j** $16c^2+40cd+25d^2$, **k** $49g^2-42gh+9h^2$, **l** $144j^2+120jk+25k^2$, **m** $p^2q^2+2pqr+r^2$, **n** $36a^2b^2-84abcd+49c^2d^2$, **o** $x^2+2+\frac{1}{x^2}$ **10 a** no, **b** yes, **c** no, **d** yes, **e** yes, **f** no, **g** no, **h** no, **i** yes **11 a** 25, **b** $6p$, **c** $y+6$, $12y$, **d** $a-7$, $14a$, **e** $k+4$, 16, **f** $u-1$, 1, **g** $12b$, **h** $3e+5$, $30e$, **i** $5n+6$, 36, **j** $7z-3$, 9 **12 a** m^2-n^2, **b** p^2-4, **c** r^2-25, **d** $9-g^2$, **e** y^2-49, **f** $16-w^2$, **g** t^2-1, **h** k^2-36, **i** $100-b^2$, **j** $4e^2-9$, **k** $25r^2-4$, **l** $9c^2-1$, **m** $16-49h^2$, **n** $36-25n^2$, **o** $4p^2-81q^2$, **p** $100y^2-9z^2$, **q** $49s^2-36t^2$, **r** $a^2b^2-64c^2$ **13 a** $a^2+9a+18$, **b** t^2-46, **c** $2p^2-3p+6$, **d** $4e^2-8e+10$, **e** $a^2+8a+35$, **f** $2x^2+12x+25$, **g** $-6m-25$, **h** $14y+98$, **i** $k^2-21k+14$, **j** $120c$, **k** $32h+64$, **l** $24m-18$ **14 a** $x^3+5x^2+10x+8$, **b** $a^3-6a^2+13a-20$, **c** $k^3+6k^2+11k+6$, **d** $n^3-6n^2-13n+42$, **e** $p^3+15p^2+63p+49$, **f** $y^3-11y^2+39y-45$, **g** $t^3+6t^2+12t+8$, **h** $e^3-9e^2+27e-27$

Exercise 5.2

1 a $2(c+3)$, **b** $5(m+4)$, **c** $3(7+e)$, **d** $7(7+x)$, **e** $3(g-5)$, **f** $6(k-1)$, **g** $3(9-r)$, **h** $5(11-t)$, **i** $2(x+y)$, **j** $3(m-4n)$, **k** $8(3p+q)$, **l** $12(f-4g)$, **m** $x(y+z)$, **n** $b(a-c)$, **o** $m(m+7)$, **p** $c(c-1)$ **2 a** $2(2n+3)$, **b** $3(3b+5)$, **c** $4(3y-2)$, **d** $5(2u-5)$, **e** $7(3p+4)$, **f** $3(5g-6)$, **g** $7(2w+3)$, **h** $6(4z-5)$, **i** $11(4h-3)$, **j** $7(5d+7)$, **k** $10(2q-5)$, **l** $11(4f-7)$, **m** $5(5-9k)$, **n** $7(8+9v)$, **o** $8(5-8a)$, **p** $12(2-7s)$ **3 a** $3y(x+4z)$, **b** $2p(q+5r)$, **c** $5g(f-3h)$, **d** $7c(d-6e)$, **e** $3j(2i+3k)$, **f** $4n(3m-5p)$, **g** $11v(2u-3w)$, **h** $8t(4s+3u)$, **i** $6b(b+4)$, **j** $7a(3a-1)$, **k** $2q(7q-10)$, **l** $5u(5+6u)$, **m** $pq(r+s)$, **n** $de(c-f)$, **o** $xy(x+y)$, **p** $ab(c-b)$, **q** $fg(g-fh)$, **r** $3k(2j+5km)$, **s** $4tu(4u+5tv)$, **t** $7gh(4gh-7i)$ **4 a** $4(p+q+r)$, **b** $a(b+c-d)$, **c** $x(x-y+z)$, **d** $3(e+2f+5g)$, **e** $2(3m^2-5m+7n)$, **f** $5(1-3v-6v^2)$, **g** $6(5c^2-2c+3)$, **h** $a(11b-1+b^2)$, **i** $3r(r-s-3)$, **j** $5y(4-5z+2y)$, **k** $7i(j+2i-6k)$, **l** $12z(2-5y-8z)$, **m** $pq(p+1+q)$, **n** $rs(9-s-r)$, **o** $abc(a+b-c)$ **5 a** $-2(k+4)$, **b** $-3(n+5)$, **c** $-5(2c+5)$, **d** $-7(2w+3)$, **e** $-4(x-1)$, **f** $-9(d-2)$, **g** $-4(4m-5)$, **h** $-3(3g-7)$, **i** $-9(2+3y)$, **j** $-3(12-5e)$, **k** $-8(4+5z)$, **l** $-11(4-9t)$, **m** $-d(c+e)$, **n** $-j(i-k)$, **o** $-a(a+7)$, **p** $-v(11-v)$, **q** $-4n(2n-5)$, **r** $-3b(7+8b)$, **s** $-4f(5-9e)$, **t** $-13cd(2c+3d)$ **6 a** $m^2(1+m)$, **b** $x^2(x^4+1)$, **c** $t^3(1-t^5)$, **d** $y^3(y^7-1)$, **e** $a^4(1+3a^3)$, **f** $g^3(2-g^5)$, **g** $u^5(7u^4+4)$, **h** $h^3(5-4h)$, **i** $3c^5(c^5+3)$, **j** $11g^3(2g^5+1)$, **k** $8q^7(2-3q^6)$, **l** $5z^2(9z^7-4)$

Exercise 5.3

1 a x^2-9, $(x+3)(x-3)$, **b** $4x^2-25$, $(2x+5)(2x-5)$ **2 a** $(p-q)(p+q)$, **b** $(c-d)(c+d)$, **c** $(m-n)(m+n)$, **d** $(u-v)(u+v)$ **3 a** $(x-2)(x+2)$, **b** $(a-5)(a+5)$, **c** $(p-1)(p+1)$, **d** $(y-6)(y+6)$, **e** $(z-8)(z+8)$, **f** $(c-11)(c+11)$, **g** $(t-7)(t+7)$, **h** $(b-9)(b+9)$, **i** $(4-k)(4+k)$, **j** $(8-g)(8+g)$, **k** $(10-m)(10+m)$, **l** $(12-u)(12+u)$ **4 a** $(e-13)(e+13)$, **b** $(h-16)(h+16)$, **c** $(23-s)(23+s)$, **d** $(19-j)(19+j)$ **5 a** $(2a-3)(2a+3)$, **b** $(3p-5)(3p+5)$, **c** $(7q-2)(7q+2)$, **d** $(4c-1)(4c+1)$, **e** $(6-7y)(6+7y)$, **f** $(5-4r)(5+4r)$, **g** $(1-11u)(1+11u)$, **h** $(10-9t)(10+9t)$, **i** $(3a-b)(3a+b)$, **j** $(x-2y)(x+2y)$, **k** $(6e-f)(6e+f)$, **l** $(j-8k)(j+8k)$, **m** $(2g-7h)(2g+7h)$, **n** $(5m-12n)(5m+12n)$, **o** $(10p-3q)(10p+3q)$, **p** $(6s-11t)(6s+11t)$, **q** $(ab-c)(ab+c)$, **r** $(p-qr)(p+qr)$, **s** $(2x-3yz)(2x+3yz)$, **t** $(4ef-9gh)(4ef+9gh)$ **6 a** $2(m-3)(m+3)$, **b** $3(a-2)(a+2)$, **c** $11(t-1)(t+1)$, **d** $5(y-10)(y+10)$, **e** $3(5-x)(5+x)$, **f** $7(3-p)(3+p)$, **g** $2(8-e)(8+e)$, **h** $6(3-z)(3+z)$, **i** $2(2n-3)(2n+3)$, **j** $3(2c-3)(2c+3)$, **k** $5(3-2f)(3+2f)$, **l** $2(3k-5)(3k+5)$, **m** $a(a-1)(a+1)$, **n** $n^2(1-n)(1+n)$, **o** $d(7d-1)(7d+1)$, **p** $u(4-5u)(4+5u)$, **q** $8h(h-2)(h+2)$, **r** $6w(3w-2)(3w+2)$, **s** $8s(3-2s)(3+2s)$, **t** $5j(4j-5)(4j+5)$ **7 a** $(2a-6)(2a+6)$, no — each factor still has a common factor of 2. **b** $4(a-3)(a+3)$ **8 a** $9(k-2)(k+2)$, **b** $4(c-5)(c+5)$, **c** $25(x-2y)(x+2y)$, **d** $4(e-6f)(e+6f)$ **9** 31 **10 a** 240, **b** 9800, **c** 2000 **11 a** $(a+b-c)(a+b+c)$, **b** $(m-n-p)(m-n+p)$, **c** $(x+3-y)(x+3+y)$, **d** $(j-k-6)(j-k+6)$, **e** $(b-2)(b+12)$, **f** $p(p+4)$, **g** $(2m+2n-3)(2m+2n+3)$, **h** $(3c-3d-5)(3c-3d+5)$, **i** $(27p+18q-8r)(27p+18q+8r)$

ANSWERS

Exercise 5.4

1 **a** $(c+d)(a+b)$, **b** $(x+3)(x+2)$, **c** $(p-4)(3n+7)$, **d** $(x+y)(w-z)$, **e** $(t-1)(t-9)$, **f** $(a-8)(4g-5h)$, **g** $(q+r)(p+s)$, **h** $(a^2+2)(5-c)$, **i** $(y+4)(mn-8)$, **j** $(d+e)(u+1)$, **k** $(p-2q)(1+w)$, **l** $(7i-2j)(6h-1)$, **2** **a** $(m+n)(k+5)$, **b** $(c+d)(a+b)$, **c** $(p+q)(p+2)$, **d** $(w+3)(x+2y)$, **e** $(e-2)(c+4d)$, **f** $(h-1)(2g+3)$, **g** $(v+7)(4u+5)$, **h** $(x+2y)(3+8a)$, **i** $(n-3)(5m+6p)$, **j** $(k+6h)(3g+4)$, **k** $(r+6)(pq+7)$, **l** $(m+p)(n+1)$, **m** $(a-5b)(1+c)$, **n** $(3e+f)(1+e)$, **o** $(a+1)(a^2+1)$ **3** **a** $(c+d)(2-e)$, **b** $(q+s)(p-r)$, **c** $(p-q)(p-8)$, **d** $(y-3)(x-y)$, **e** $(h+i)(g-1)$, **f** $(3u-2v)(1-w)$, **g** $(x-4)(x^2-y)$, **h** $(k+7)(k-2m)$, **i** $(m-4)(jk-2)$, **j** $(5p-2q)(7n-1)$, **k** $(z-w)(3z-u)$, **l** $(2x+3y)(5-2z)$ **4** **a** $(y+z)(x+w)$, **b** $(p+q)(r+2)$, **c** $(n+p)(3m+k)$, **d** $(4y+3)(z+5)$, **e** $(d+7)(c+1)$, **f** $(e+1)(5f+6)$ **5** **a** $(2-y)(x-y)$, **b** $(z-7)(y-z)$, **c** $(b-c)(a+b)$, **d** $(a-b)(a^2-b)$, **e** $(11-n)(m+n)$, **f** $(m-4n)(1+5p)$, **g** $(v-w)(6u+5w)$, **h** $(2d-c)(7e-1)$, **i** $(x-y)(2w-y)$, **j** $(c-d)(c^2+d)$, **k** $(2q-5r)(5p-6q)$, **l** $(2s-7t)(9r+4s)$ **6** **a** $(x+4)(x+3+y)$, **b** $(m-2)(m-5+n)$, **c** $(k-4)(k+6-m)$, **d** $(a+b)(a+b+c)$, **e** $(x-7)(x-7-w)$, **f** $(g+h)(f+g+h)$ **7** **a** $(2c-d)(c+3d+5)$, **b** $(p-q)(p-q+3r)$

Exercise 5.5

1 **a** 3, 5, **b** 2, 7, **c** 4, 6, **d** 3, 10, **e** –2, –5, **f** –1, –2, **g** –4, –7, **h** –8, –9, **i** 6, –2, **j** 5, –4, **k** –7, 5, **l** –9, 6
2 **b** **i** $a>0, b>0$, **ii** $a<0, b<0$, **iii** $a>0, b<0$, **iv** $a<0, b>0$ **3** **a** $(x+2)(x+3)$, **b** $(y+3)(y+4)$, **c** $(u+1)(u+2)$, **d** $(m+4)(m+6)$, **e** $(a+3)(a+7)$, **f** $(t+3)(t+6)$, **g** $(k+2)(k+7)$, **h** $(p+1)(p+7)$, **i** $(n+5)(n+9)$, **j** $(d+4)(d+8)$, **k** $(s+5)(s+6)$, **l** $(b+4)(b+9)$, **m** $(e+5)(e+10)$, **n** $(c+5)(c+12)$, **o** $(r+8)(r+11)$, **p** $(z+8)(z+12)$ **4** **a** $(m-3)(m-5)$, **b** $(q-2)(q-5)$, **c** $(d-2)(d-10)$, **d** $(a-5)(a-7)$, **e** $(u-2)(u-9)$, **f** $(e-1)(e-8)$, **g** $(n-5)(n-8)$, **h** $(w-4)(w-5)$, **i** $(h-7)(h-9)$, **j** $(v-3)(v-8)$, **k** $(t-1)(t-6)$, **l** $(s-4)(s-11)$, **m** $(k-3)(k-10)$, **n** $(j-7)(j-11)$, **o** $(x-6)(x-9)$, **p** $(f-6)(f-8)$ **5** **a** $(y+6)(y-2)$, **b** $(d+5)(d-3)$, **c** $(a+4)(a-1)$, **d** $(p+8)(p-2)$, **e** $(v-5)(v+2)$, **f** $(u-9)(u+2)$, **g** $(m-6)(m+5)$, **h** $(x-5)(x+1)$, **i** $(f+7)(f-3)$, **j** $(w-9)(w+7)$, **k** $(k-9)(k+3)$, **l** $(c+8)(c-4)$, **m** $(z-8)(z+5)$, **n** $(i+4)(i-3)$, **o** $(r-11)(r+9)$, **p** $(e-7)(e+2)$, **q** $(s+10)(s-7)$, **r** $(h+8)(h-7)$, **s** $(b-10)(b+4)$, **t** $(t+12)(t-9)$ **6** **a** $(n-2)(n-4)$, **b** $(c+6)(c+7)$, **c** $(x-2)(x+1)$, **d** $(d+11)(d-4)$, **e** $(q+2)(q+11)$, **f** $(t-1)(t-10)$, **g** $(v+6)(v+8)$, **h** $(j-12)(j+6)$, **i** $(g-10)(g+2)$, **j** $(b+3)(b+12)$, **k** $(r-9)(r+6)$, **l** $(u-1)(u-12)$, **m** $(e+10)(e+12)$, **n** $(l-7)(l+6)$, **o** $(y-2)(y-12)$, **p** $(p+11)(p-5)$, **q** $(z-10)(z+8)$, **r** $(a+10)(a-6)$, **s** $(f-3)(f-13)$, **t** $(m+10)(m+11)$, **u** $(w-9)(w+4)$, **v** $(k+2)(k+13)$, **w** $(h-7)(h-12)$, **x** $(i+12)(i-11)$ **7** **a** $(p+3)^2$, **b** $(c+5)^2$, **c** $(g-4)^2$, **d** $(y-7)^2$, **e** $(t+1)^2$, **f** $(r-6)^2$, **g** $(x-9)^2$, **h** $(j+11)^2$ **8** **a** $2(m-2)(m-8)$, **b** $3(k+2)(k+6)$, **c** $4(a+8)(a-2)$, **d** $5(c-9)(c+3)$, **e** $3(t-1)(t-8)$, **f** $7(x-4)(x+3)$, **g** $4(d-3)(d-5)$, **h** $5(n+10)(n-2)$, **i** $10(y-11)(y+3)$
9 **a** $x+6$, **b** $g+5$ **10** **a** $(x-1)(x+1)(x-2)(x+2)$, **b** $(x-2)(x+2)(x-5)(x+5)$, **c** $(x^2-2)(x-3)(x+3)$, **d** $(x^2+5)(x-2)(x+2)$, **e** $(x^2+8)(x-1)(x+1)$, **f** $(x-1)(x+1)(x-3)(x+3)$, **g** $(x^2-3)(x-4)(x+4)$, **h** $(x-2)(x+2)(x-3)(x+3)$, **i** $(x-1)(x+1)(x-4)(x+4)$, **j** $(x^2-6)(x-1)(x+1)$, **k** $(x^2+7)(x-2)(x+2)$, **l** $(x-3)(x+3)(x-4)(x+4)$

Exercise 5.6

1 B **2** C **3** D **4** A **5** **a** $(2x+1)(x+3)$, **b** $(3x+2)(x+4)$, **c** $(4x+7)(x+1)$, **d** $(2x+1)(x+5)$, **e** $(5x+2)(x+4)$, **f** $(7x+1)(x+5)$, **g** $(2x+9)(x+6)$, **h** $(7x+6)(x+3)$, **i** $(3x+8)(x+4)$, **j** $(2x-1)(x-2)$, **k** $(3x-2)(x-3)$, **l** $(5x-2)(x-3)$, **m** $(2x-3)(x-5)$, **n** $(3x-4)(x-4)$, **o** $(5x-6)(x-2)$, **p** $(3x-5)(x-7)$, **q** $(2x-11)(x-4)$, **r** $(7x-6)(x-5)$ **6** **a** $(3x-1)(x+2)$, **b** $(2x+5)(x-2)$, **c** $(2x-3)(x+5)$, **d** $(3x-7)(x+3)$, **e** $(2x+5)(x-5)$, **f** $(3x+2)(x-8)$, **g** $(5x+9)(x-4)$, **h** $(11x+3)(x-5)$, **i** $(4x-1)(x+7)$, **j** $(6x+1)(x-3)$, **k** $(5x-6)(x+2)$, **l** $(4x+7)(x-6)$, **m** $(7x-5)(x+8)$, **n** $(7x-6)(x+6)$, **o** $(4x-7)(x+4)$, **7** **a** $(2k+5)(k+8)$, **b** $(2c+9)(c-3)$, **c** $(4n-9)(n-2)$, **d** $(3y+4)(y-7)$, **e** $(7p-3)(p-4)$, **f** $(6a+1)(a-4)$, **g** $(4b+7)(b+6)$, **h** $(5u-12)(u+3)$, **i** $(2w+11)(w-10)$, **j** $(5h+4)(h+3)$, **k** $(3j-10)(j+4)$, **l** $(6l-5)(l-9)$ **8** **a** $(3a+2)(2a+3)$, **b** $(2n-7)(2n-3)$, **c** $(4k+9)(2k-1)$, **d** $(3p-5)(3p+2)$, **e** $(3c+5)(2c+3)$, **f** $(2e-5)(2e+9)$, **g** $(2t-3)(6t-5)$, **h** $(5b-8)(2b+9)$, **i** $(3m-8)(4m-5)$, **j** $(5y+2)(4y+5)$, **k** $(9w+11)(3w-5)$, **l** $(5q-2)(4q+3)$ **9** **a** $(7-x)(4+x)$, **b** $(5-2a)(3+a)$, **c** $(4-3p)(6+p)$, **d** $(5-6m)(3-2m)$, **e** $(7+5g)(3+4g)$ **f** $(7-2w)(8+5w)$
10 **a** $2(2k+5)(k+2)$, **b** $3(3p-1)(2p-3)$, **c** $2(5a+3)(2a-1)$, **d** $3(2v+3)(2v-7)$, **e** $4(3f-1)(2f-3)$, **f** $5(2e-5)(e+4)$ **11** **a** $3a+4$, **b** $2n+9$ **12** **a** $(3x+1)(x+3)$, **b** $(2x-3)(x-4)$, **c** $(3x-2)(x+2)$, **d** $(4x+3)(x-5)$, **e** $(7x-1)(x-2)$, **f** $(5x-3)(x+5)$

Exercise 5.7

1 **a** $10(e+3)$, **b** $(a-2)(a+2)$, **c** $(m+3)(m+4)$, **d** $(n+6)(n+p)$, **e** $(2x+5)(x+4)$, **f** $(5-k)(5+k)$, **g** $x(x-1)$, **h** $(t-2)(t-4)$, **i** $(w+2)^2$, **j** $(p-3)(a+5)$, **k** $5(3y+7)$, **l** $(2q-1)(q-3)$, **m** $(g-h)(g+h)$, **n** $-7(j-2k)$, **o** $(u-5)(u+3)$, **p** $(3b+2)(b+8)$, **q** $7t(4t-9u)$, **r** $(2c-3)(2c+3)$, **s** $(y-2)(4x+9)$, **t** $fg(e+h)$, **u** $(h+10)(h-2)$, **v** $(5v-6w)(5v+6w)$, **w** $(g-6)(5m-2)$, **x** $(2f-5)(f+1)$ **2** **a** $9p(3q+5p)$, **b** $(x-11)(x+7)$, **c** $(3e-8f)(3e+8f)$, **d** $(3b+11)(b-4)$, **e** $(g+3h)(5-2i)$, **f** $12y(5x-7)$, **g** $(z+15)(z-2)$, **h** $(pq-r)(pq+r)$, **i** $(8-k)(5+2k)$, **j** $(8-kl)(8+kl)$, **k** $(m+7)(m^2-n)$, **l** $(5s-4)(2s-9)$, **m** $9(h-2)(h+2)$, **n** $3(x+5)(x-4)$, **o** $(5-3j)(1-4k)$,

p $n(n-1)(n+1)$, **q** $(a+b)(a+b+c)$, **r** $(1+t)(1+t^2)$, **s** $2(u+5)^2$, **t** $-2(f-4)(f+4)$, **u** $(2a-5b)(5+6c)$, **v** $a(10a+14b-1)$, **w** $2k(k+9)(k-3)$, **x** $v(4u+7)(u-6)$ **3 a** $(x+y)(x+y+z)$, **b** $(t^2+1)(t-1)(t+1)$, **c** $(w-1)(w+1)(w-5)(w+5)$, **d** $25b^2(a-2c)(a+2c)$, **e** $(4+n^2)(2-n)(2+n)$, **f** $(y+18)(y-4)$, **g** $(a+b+c+d)(a+b-c-d)$, **h** $(a-3)(a+3)(a-b)$, **i** $(x-5)(x+5)(4g+3)$

Exercise 5.8

1 a $\frac{a}{2}$, **b** $\frac{t}{5}$, **c** $\frac{3k}{4}$, **d** $\frac{4m}{5}$, **e** $\frac{3y}{2}$, **f** $\frac{4c}{3}$, **g** $\frac{1}{d}$, **h** $\frac{1}{4g}$, **i** $\frac{a}{c}$, **j** $\frac{q}{5}$, **k** $\frac{p}{s}$, **l** $\frac{b}{3}$, **m** $\frac{5g}{6}$, **n** $\frac{6}{7y}$, **o** $\frac{4e}{3g}$, **p** u, **q** $\frac{1}{s}$, **r** $\frac{v}{w}$, **s** $\frac{4r}{5}$, **t** $\frac{12d}{7c}$ **2 a** $x+1$, **b** $k-2$, **c** $2m+3$, **d** $\frac{3a-4}{2}$, **e** $\frac{4t-5}{2}$, **f** $b+c$, **g** $\frac{1}{v-w}$, **h** $x+6$, **i** $\frac{1}{2-5t}$, **j** $\frac{2c-d}{3}$, **k** $\frac{n+2m}{3p}$, **l** $\frac{3g-4h}{11g}$, **m** $\frac{1}{4}$, **n** $\frac{x}{z}$, **o** $\frac{2}{3}$, **p** $\frac{1}{3}$ **3 a** $m+7$, **b** $\frac{a-5}{4}$, **c** $\frac{x}{x+1}$, **d** $\frac{1}{n-12}$, **e** $3t+5$, **f** $h+1$, **g** $\frac{3}{2(k-3)}$, **h** $\frac{p-q}{p+q}$ **4 a** $x+1$, **b** $a-4$, **c** $\frac{w+4}{2}$, **d** $\frac{1}{n-5}$, **e** $\frac{c-3}{5c}$, **f** $\frac{e+3}{e-11}$, **g** $\frac{z-9}{z-10}$, **h** $\frac{s-4}{s+11}$, **i** $\frac{d+8}{2(d-3)}$ **5 a** $2x+5$, **b** $\frac{1}{5r-2}$, **c** $\frac{2p+3}{7}$, **d** $\frac{5k+8}{k+4}$, **e** $\frac{4h+9}{h-6}$, **f** $\frac{3v+2}{4v-1}$ **6 a** $p+q$, **b** $\frac{k+3}{m+5}$, **c** $\frac{u+7}{v-w}$, **d** $\frac{x+y}{x-3}$, **e** $\frac{b-c}{2(a+4)}$, **f** $\frac{n-p}{n+p}$ **7 a** -1, **b** -8, **c** $-\frac{1}{5}$, **d** $-\frac{2}{3}$, **e** $-a-13$, **f** $\frac{6-c}{2}$, **g** $\frac{9-s}{3}$, **h** $\frac{3-g}{g^2}$, **i** $\frac{-3(4+k)}{k+2}$

Exercise 5.9

1 a $\frac{xy}{14}$, **b** $\frac{4}{9}$, **c** $\frac{9c}{10}$, **d** $\frac{5eq}{12cp}$, **e** $\frac{mn}{12}$, **f** $\frac{3}{2y}$, **g** $\frac{4s}{9t}$, **h** $\frac{66d}{49c}$ **2 a** 4, **b** $\frac{2}{3}$, **c** $\frac{2(b-4)}{3}$, **d** $\frac{8}{9}$, **e** $\frac{2(x-y)}{21}$, **f** $\frac{3(k+2)}{4(k+1)}$ **3 a** $\frac{2}{5}$, **b** $\frac{3}{7}$, **c** $\frac{ac}{c-3}$, **d** $\frac{c(a+b)^2}{9}$, **e** $\frac{25u}{12t}$, **f** $\frac{8m}{21n}$ **4 a** $\frac{a+3}{5}$, **b** $\frac{s}{7}$, **c** $\frac{3t}{2(t-4)}$, **d** $\frac{9x}{x-5}$, **e** $\frac{5}{k}$, **f** $\frac{c+2}{c+7}$, **g** $\frac{n+1}{n-1}$, **h** $\frac{r+7}{5r}$, **i** $\frac{a+b}{ab}$, **j** $\frac{b(b-4)}{2}$, **k** $\frac{(w+11)^2}{24w}$, **l** $\frac{a+2}{a}$, **m** $\frac{e+4}{3e}$, **n** $\frac{c+d}{d}$ **5 a** $\frac{7}{k}$, **b** $\frac{n+2}{n+8}$, **c** $\frac{1}{2}$, **d** $\frac{12(a-1)}{5}$, **e** $\frac{z+10}{z}$, **f** $\frac{2}{7}$, **g** $\frac{1}{4}$, **h** $\frac{r-8}{r-7}$, **i** $\frac{m+3}{2m}$, **j** $\frac{3x}{x-1}$, **k** $\frac{q-2}{q+8}$, **l** $\frac{1}{2v}$, **m** 2, **n** $\frac{7c+11}{2c-1}$ **6 a** $-\frac{8}{15}$, **b** $-\frac{n}{9}$, **c** $-\frac{4}{15xz}$, **d** $\frac{1}{q^2}$

Exercise 5.10

1 a $\frac{2z}{3}$, **b** $\frac{3a}{2}$, **c** $\frac{m}{2}$, **d** $\frac{x}{2}$, **e** $\frac{5k}{12}$, **f** $\frac{29f}{40}$, **g** $\frac{7}{4y}$, **h** $\frac{11}{9u}$ **2 a** $\frac{3x+7}{8}$, **b** $\frac{9n-18}{14}$, **c** $\frac{9t+14}{12}$, **d** $\frac{4b+13}{10}$, **e** $\frac{k+31}{12}$, **f** $\frac{5z-9}{24}$ **3 a** $\frac{2x+1}{x(x+1)}$, **b** $\frac{2x}{(x+1)(x-1)}$, **c** $\frac{2x+5}{(x+2)(x+3)}$, **d** $\frac{5x+15}{x(x+5)}$, **e** $\frac{5x+13}{(x+3)(x+2)}$, **f** $\frac{7x+28}{(x-1)(x+6)}$, **g** $\frac{1}{x(x+1)}$, **h** $\frac{-2}{x(x+2)}$, **i** $\frac{2}{(x-1)(x+1)}$, **j** $\frac{2x+8}{x(x+2)}$, **k** $\frac{3x+1}{(x+5)(x+3)}$, **l** $\frac{9x-17}{(2x-1)(3x-4)}$ **4 a** $\frac{x^2+3x+1}{(x+1)(x+2)}$, **b** $\frac{x^2-5x}{(x+3)(x-1)}$, **c** $\frac{2x^2+7x+7}{(x+3)(x+1)}$ **5 a** $\frac{1}{x}$, **b** $\frac{x+2}{(x+1)(x+5)}$, **c** $\frac{2x+6}{(x+1)(x+2)(x+5)}$, **d** $\frac{2x+9}{(x+4)(x-2)(x+5)}$, **e** $\frac{6}{x(x-4)(x+6)}$, **f** $\frac{3}{(x-1)(x+1)(x+2)}$, **g** $\frac{5x+7}{(x+2)(x+4)(x+1)}$ **h** $\frac{9x+13}{(x-3)(x+2)(x+5)}$, **i** $\frac{5x-28}{2x(2x-1)(x-4)}$, **j** $\frac{20-2x}{(3x+2)(x-2)(x+6)}$, **k** $\frac{3x^2-5x}{(x-3)(x-4)(x+1)}$ **l** $\frac{2x^2+3x-1}{x(x-1)(x+1)}$ **6 a** $\frac{x+2}{2x(x+4)}$, **b** $\frac{7}{12(x+3)}$, **c** $\frac{x+5}{(x-2)(x+2)}$, **d** $\frac{2x+26}{5(x-3)(x+3)}$, **e** $\frac{2x+1}{(x-1)(x+1)(x+2)}$, **f** $\frac{2x+5}{(x-1)(x-6)(x+6)}$, **g** $\frac{2x+9}{(x+3)(x+5)(x+6)}$, **h** $\frac{2x+16}{(x-9)(x+7)(x+9)}$,

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i $\frac{7x+14}{(x+5)(x-5)(x-2)}$, **j** $\frac{7x+7}{(x-10)(x+3)(x-4)}$, **k** $\frac{19x+27}{(x-3)(x+3)(3x+4)}$, **l** $\frac{8x+1}{(2x+5)(x+7)(x-7)}$

7 a $\frac{3-x}{3x(x-6)}$, **b** $\frac{3}{10(x+4)}$, **c** $\frac{9-2x}{(x-4)(x+4)}$, **d** $\frac{-4x-17}{5(x-8)(x+8)}$, **e** $\frac{3-3x}{7(3x-4)(3x+4)}$,

f $\frac{2}{(x-7)(x+5)(x-5)}$, **g** $\frac{-1}{(x-2)(x-11)(x-3)}$, **h** $\frac{6-6x}{7x(x-2)(x+6)}$, **i** $\frac{30-x}{(x-12)(x+12)(x-3)}$,

j $\frac{3x-25}{(x-10)(x+5)(x-9)}$, **k** $\frac{22x}{(2x-7)(2x+7)(x+2)}$, **l** $\frac{36-18x-3x^2}{(4x-3)(x+10)(x-10)}$

Chapter 5 Review

1 a $y^2+9y+20$, **b** $m^2-10m+21$, **c** $t^2+6t-16$, **d** $a^2-7a-44$ **2 a** $6p^2-25p+24$, **b** $8-2r-3r^2$
3 a $m^2-14m+49$, **b** $4c^2+20c+25$, **4** $162x^3-72x^2y+8xy^2$ **5 a** $(t+11)^2=t^2+22t+121$,
b $(n-9)^2=n^2-18n+81$, **c** $(3x+5)^2=9x^2+30x+25$, **d** $(4u-11)^2=16u^2-88u+121$ **6 a** no, **b** no, **c** yes,
d no, **e** no, **f** yes **7 a** $a^3+8a^2+19a+12$, **b** $10n^2-16n+28$, **c** $14a-59$, **d** $u^2-2u-18$ **8 a** $7(x+4)$,
b $a(a+1)$, **c** $5(2m+3)$, **d** $3(3p-8q)$, **e** $4s(3r+5t)$, **f** $pq(r-s)$, **g** $9y(3y-4)$, **h** $gh(g-h)$, **i** $-a(b-3)$,
j $-2c(9c+7d)$ **9 a** $(m-n)(m+n)$, **b** $(z-3)(z+3)$, **c** $(1-p)(1+p)$, **d** $(7-r)(7+r)$, **e** $(w-23)(w+23)$,
f $(2a-5)(2a+5)$, **g** $(x-4y)(x+4y)$, **h** $(9u-10v)(9u+10v)$, **i** $(ab-c)(ab+c)$, **j** $(6pq-11rs)(6pq+11rs)$
10 a $(z+3)(x+y)$, **b** $(q-r)(p-1)$, **c** $(e+f)(e-2)$, **d** $(3k-7)(2j-5)$, **e** $(4-q)(p-q)$, **f** $(10-d)(c+d)$
11 a $(x+1)(x+3)$, **b** $(b-3)(b-7)$, **c** $(e-4)(e+3)$, **d** $(p+8)(p-5)$, **e** $(a+5)^2$, **f** $(q-9)^2$ **12 a** $(3t+2)(t+4)$,
b $(2m-7)(m-3)$, **c** $(3c-2)(c+3)$, **d** $(7b+5)(b-4)$, **e** $(3s+5)(2s+7)$, **f** $(4d-9)(3d+2)$ **13 a** $(n-6)(n+6)$,
b $(v+3)(v+5)$, **c** $5(4k+7)$, **d** $(3e+5)(e+3)$, **e** $(a+4)(a+b)$, **f** $(3h-5)(3h+5)$, **g** $(p-8)(p+3)$,
h $-7x(3y+5z)$, **i** $(1+u)(1+u^2)$, **j** $(4a-11b)(4a+11b)$, **k** $(5m+6)(m-4)$, **l** $(y-10)(y+1)$, **m** $(d+3e)(2c-5)$,
n $rs(s-r+1)$, **o** $(6g+5)(5g-3)$ **14 a** $3(x-2)(x+2)$, **b** $2(a+3)(a+6)$, **c** $n(n-1)(n+1)$, **d** $4(h-3)(h+3)$,
e $a(b-3)(b-4)$, **f** $2(3u+1)(u+4)$, **g** $3(2z-5)(2z+5)$, **h** $x(x-2y)(x-3)$ **15 a** $\frac{1}{d}$, **b** $\frac{3t}{4v}$, **c** $\frac{5a}{2}$, **d** $\frac{p}{q}$

16 a $3e+5$, **b** $\frac{1}{a+9}$, **c** $\frac{3x-5y}{2}$, **d** $\frac{2}{3}$, **e** $u+2$, **f** $\frac{4h-5}{2}$, **g** $\frac{x+5}{x+3}$, **h** $\frac{k+7}{k+4}$, **i** $\frac{3p-4}{p-5}$, **j** $\frac{b-c}{2a-7}$, **k** -5,
l $\frac{-c-8}{3}$ **17 a** $\frac{2a}{a-3}$, **b** $\frac{c-4}{c}$, **c** $\frac{y}{2}$, **d** $\frac{x}{2x+1}$ **18 a** $\frac{7a+26}{12}$, **b** $\frac{h-13}{10}$, **c** $\frac{2x+3}{x(x+3)}$, **d** $\frac{x^2-x+2}{(x-1)(x+1)}$,
e $\frac{2x^2+9x+11}{(x+2)(x+1)}$ **19 a** $\frac{x+1}{3(x+2)(x-2)}$, **b** $\frac{2x+12}{(x-5)(x+5)(x+7)}$, **c** $\frac{x-5}{(x-2)(x-9)(x-3)}$,
d $\frac{x^2-3x+12}{(2x-3)(2x+3)(x+5)}$

6 Quadratic equations

Exercise 6.1

1 a i 17, **ii** 10, **iii** 1, **b i** 50, **ii** 8, **iii** 4.5, **c i** -3, **ii** -3, **iii** $\frac{3}{4}$, **d i** 20, **ii** 8, **iii** -28, **e i** 28, **ii** 10, **iii** $5\frac{5}{16}$,
f i 19, **ii** $9\frac{5}{9}$, **iii** $15\frac{18}{25}$ **2 a** no, **b** yes, **c** yes, **d** no, **e** yes, **f** yes, **g** yes, **h** no, **i** yes, **j** no, **k** no, **l** yes, **m** no,
n yes, **o** yes, **p** no **3 a** $y=x^2+1$, **b** $y=x^2-3$, **c** $y=2x^2$, **d** $y=-3x^2$, **e** $y=2x^2+5$, **f** $y=3x^2-2$
4 a $y=x^2+2x+3$, **b** $y=x^2+6x+2$, **c** $y=x^2+4x-1$, **d** $y=x^2-5x+8$, **e** $y=x^2+8x-10$, **f** $y=x^2-x-7$,
g $y=x^2+3x+11$, **h** $y=x^2-7x+20$ **5 a** $x=3$, **b** $x=4$, **c** $x=7$, **d** $x=5$, **e** $x=6$, **f** $x=12$ **6 a** $x=-1, 3$,
b $x=-5, 4$, **c** $x=-4, 7$ **7 a** $x=3, 4$, **b** $x=-1, 4$, **c** $x=-5, 3$ **8** 14 **9 a** $(x+7)$ cm, **b** $x=8$, **c** 15 cm × 8 cm

Exercise 6.2

1 a $x=0, 2$, **b** $a=0, -5$, **c** $t=0, 4$, **d** $m=0, 1$, **e** $k=0, -3$, **f** $c=0, 6$, **g** $x=1, 2$, **h** $e=-5, -3$, **i** $p=-2, 4$,
j $n=7, -1$, **k** $z=-8, -6$, **l** $q=11, -11$, **m** $w=0, \frac{1}{2}$, **n** $f=0, \frac{2}{3}$, **o** $u=0, -\frac{3}{5}$, **p** $a=\frac{1}{2}, 4$, **q** $q=-\frac{3}{4}, 1$, **r** $c=\frac{5}{3}, -6$,
s $t=\frac{2}{5}, \frac{5}{2}$, **t** $v=-\frac{7}{3}, -\frac{3}{2}$, **u** $b=\frac{4}{9}, -\frac{2}{11}$, **v** $x=3$, **w** $y=-9$, **x** $m=\frac{7}{4}$ **2 a** $a=0, 3$, **b** $y=0, -7$, **c** $p=0, 1$,
d $q=0, -1$, **e** $e=0, 4$, **f** $g=0, -2$, **g** $n=-3, 3$, **h** $d=-6, 6$, **i** $r=-1, 1$, **j** $h=-9, 9$, **k** $f=-8, 8$, **l** $t=-11, 11$,
m $k=-1, -3$, **n** $m=3, 4$, **o** $x=-6, -5$, **p** $y=-5, 3$, **q** $a=7, -3$, **r** $z=-9, 8$, **s** $n=-6, 4$, **t** $j=8, -5$, **u** $r=4, 9$,

v $p = -3$, **w** $h = 5$, **x** $v = -12$ **3 a** $y = 0, 5$, **b** $c = 0, 8$, **c** $d = 0, 9$, **d** $k = 0, \frac{4}{5}$, **e** $e = 0, \frac{7}{3}$, **f** $m = 0, \frac{2}{11}$, **g** $x = 2$, **h** $u = -\frac{10}{3}$, **i** $t = \frac{5}{6}$ **4 a** $x = -\frac{1}{2}, -2$, **b** $a = -\frac{1}{3}, -4$, **c** $p = -\frac{3}{2}, -2$, **d** $u = \frac{5}{3}, 2$, **e** $y = \frac{2}{5}, 4$, **f** $c = \frac{1}{7}, 5$, **g** $e = \frac{4}{3}, -2$, **h** $k = \frac{1}{4}, -3$, **i** $m = -\frac{3}{2}, 9$, **j** $v = -\frac{6}{5}, 2$, **k** $g = \frac{4}{7}, -5$, **l** $d = -\frac{7}{4}, 4$, **m** $t = -\frac{1}{4}, -\frac{3}{2}$, **n** $h = \frac{1}{3}, \frac{2}{3}$, **o** $n = -\frac{3}{2}, 8$, **p** $w = -\frac{2}{3}, -\frac{1}{2}$, **q** $u = \frac{4}{3}, -\frac{5}{4}$, **r** $s = \frac{3}{5}, \frac{3}{2}$, **s** $a = \frac{5}{2}, -\frac{7}{2}$, **t** $j = \frac{2}{3}, \frac{8}{3}$, **u** $z = -\frac{3}{5}, \frac{5}{3}$ **5 a** $a = 0, 2$, **b** $p = 0, -4$, **c** $s = 0, 1$, **d** $q = -4, 4$, **e** $b = -7, 7$, **f** $u = -10, 10$, **g** $x = -6, 1$, **h** $u = 4, -2$, **i** $y = 4, 7$, **j** $n = 6, -2$, **k** $p = 5, 9$, **l** $r = -7, 6$, **m** $c = 11, -3$, **n** $u = -10, 7$, **o** $m = 5, 12$, **p** $z = \frac{2}{3}, 7$, **q** $w = -\frac{3}{5}, 4$, **r** $k = \frac{9}{2}, 10$ **6 a** $x^2 - 5x + 6 = 0$, **b** $x^2 - 5x + 4 = 0$, **c** $x^2 + 8x + 15 = 0$, **d** $x^2 + 10x + 24 = 0$, **e** $x^2 - 4x - 12 = 0$, **f** $x^2 + 6x - 7 = 0$, **g** $x^2 - 7x - 44 = 0$, **h** $x^2 + 9x - 36 = 0$

Exercise 6.3

1 a $9, x + 3$, **b** $16, x + 4$, **c** $36, x - 6$, **d** $1, x - 1$, **e** $49, x + 7$, **f** $25, x - 5$, **g** $121, x - 11$, **h** $64, x + 8$ **2 a** $\frac{9}{4}, x + \frac{3}{2}$, **b** $\frac{25}{4}, x + \frac{5}{2}$, **c** $\frac{49}{4}, x - \frac{7}{2}$, **d** $\frac{1}{4}, x - \frac{1}{2}$, **e** $\frac{121}{4}, x + \frac{11}{2}$, **f** $\frac{81}{4}, x - \frac{9}{2}$ **3 a** $x = -1, 5$, **b** $x = -8, 2$, **c** $x = -1, 3$, **d** $x = 0, -8$, **e** $x = -6, -8$, **f** $x = -2, 12$, **g** $x = -17, 5$, **h** $x = -3, 9$, **i** $x = 0, -16$ **4 a** $x = 1 \pm \sqrt{2}$, **b** $x = 2 \pm \sqrt{5}$, **c** $x = -4 \pm \sqrt{3}$, **d** $x = -3 \pm \sqrt{6}$, **e** $x = 7 \pm \sqrt{11}$, **f** $x = -5 \pm \sqrt{17}$, **g** $x = 1 \pm 2\sqrt{2}$, **h** $x = -3 \pm 2\sqrt{3}$, **i** $x = 2 \pm 3\sqrt{2}$, **j** $x = -5 \pm 2\sqrt{5}$, **k** $x = 4 \pm 3\sqrt{3}$, **l** $x = -7 \pm 5\sqrt{2}$ **5 a** $x = 1.45, -3.45$, **b** $x = 2.41, -0.41$, **c** $x = 4.65, -0.65$, **d** $x = 0.74, -6.74$, **e** $x = -0.39, -7.61$, **f** $x = 5.24, 0.76$, **g** $x = 10.48, -0.48$, **h** $x = 0.86, -12.86$, **i** $x = 11.74, 4.26$, **j** $x = 0.56, -3.56$, **k** $x = 4.30, 0.70$, **l** $x = 0.62, -1.62$, **m** $x = -0.81, -6.19$, **n** $x = 0.30, -3.30$, **o** $x = 9.62, -0.62$, **p** $x = 2.56, -1.56$, **q** $x = -0.85, -14.15$, **r** $x = 10.32, 0.68$ **6 a** $\frac{1}{16}, x + \frac{1}{4}$, **b** $\frac{1}{36}, x - \frac{1}{6}$, **c** $\frac{1}{100}, x + \frac{1}{10}$, **d** $\frac{1}{9}, x + \frac{1}{3}$, **e** $\frac{9}{25}, x - \frac{3}{5}$, **f** $\frac{4}{49}, x - \frac{2}{7}$, **g** $\frac{9}{16}, x + \frac{3}{4}$, **h** $\frac{25}{64}, x + \frac{5}{8}$, **i** $\frac{9}{100}, x - \frac{3}{10}$, **j** $\frac{49}{36}, x - \frac{7}{6}$ **7 a** $x = -0.13, -3.87$, **b** $x = 5.62, -0.62$, **c** $x = 2.46, 0.54$, **d** $x = 0.54, -2.29$, **e** $x = 1.79, -1.12$, **f** $x = 0.87, -2.07$

Exercise 6.4

1 a $x = -1, -2$, **b** $x = -2, -6$, **c** $x = -2, -5$, **d** $x = 3, 5$, **e** $x = -6, 2$, **f** $x = 4, 6$, **g** $x = 5, -2$, **h** $x = -8, 1$, **i** $x = -11, 3$, **j** $x = -7, 6$, **k** $x = 7, -4$, **l** $x = 4, 10$ **2 a** $x = -\frac{1}{2}, -3$, **b** $x = -\frac{3}{2}, -5$, **c** $x = -\frac{4}{3}, -2$, **d** $x = \frac{5}{2}, 2$, **e** $x = \frac{5}{3}, 3$, **f** $x = \frac{3}{4}, -1$, **g** $x = \frac{6}{5}, -2$, **h** $x = -\frac{4}{7}, 3$, **i** $x = -\frac{5}{4}, 7$ **3 a** $x = \frac{-3 \pm \sqrt{5}}{2}$, **b** $x = \frac{-5 \pm \sqrt{17}}{2}$, **c** $x = \frac{7 \pm \sqrt{33}}{2}$, **d** $x = \frac{-7 \pm \sqrt{57}}{2}$, **e** $x = \frac{9 \pm \sqrt{101}}{2}$, **f** $x = \frac{5 \pm \sqrt{37}}{2}$, **g** $x = \frac{-9 \pm \sqrt{41}}{4}$, **h** $x = \frac{5 \pm \sqrt{41}}{4}$, **i** $x = \frac{-3 \pm \sqrt{57}}{6}$, **j** $x = \frac{7 \pm \sqrt{33}}{8}$, **k** $x = \frac{5 \pm \sqrt{13}}{6}$, **l** $x = \frac{-1 \pm \sqrt{61}}{10}$ **4 a** $x = -2 \pm \sqrt{3}$, **b** $x = 3 \pm \sqrt{7}$, **c** $x = -1 \pm \sqrt{2}$, **d** $x = 2 \pm \sqrt{6}$, **e** $x = 4 \pm \sqrt{10}$, **f** $x = -2 \pm \sqrt{7}$, **g** $x = 5 \pm \sqrt{22}$, **h** $x = -3 \pm 2\sqrt{3}$, **i** $x = 5 \pm 2\sqrt{5}$, **j** $x = \frac{-3 \pm \sqrt{7}}{2}$, **k** $x = \frac{3 \pm \sqrt{3}}{3}$, **l** $x = \frac{2 \pm \sqrt{10}}{3}$, **m** $x = \frac{-2 \pm \sqrt{10}}{6}$, **n** $x = \frac{-4 \pm \sqrt{6}}{2}$, **o** $x = \frac{6 \pm \sqrt{21}}{5}$, **p** $x = \frac{1 \pm \sqrt{5}}{4}$, **q** $x = \frac{5 \pm \sqrt{10}}{3}$, **r** $x = \frac{-2 \pm \sqrt{22}}{6}$ **5 a** $x = -\frac{3}{2}$, **b** 1, **c** It is a perfect square. **6 a** There are no solutions. **b** $b^2 - 4ac \geqslant 0$ if the equation has solutions. **7 a** $5x^2 - 7x + 1 = 0$, **b** $2x^2 + 5x - 9 = 0$, **c** $3x^2 - 11x - 7 = 0$

Exercise 6.5

1 a $x = 0, 2$, **b** $x = -5, 5$, **c** $x = -3, 3$, **d** $x = 1, 5$, **e** $x = 1.65, -3.65$, **f** $x = 2.08, -1.08$, **g** $x = -1, -4$, **h** $x = 4, 5$, **i** $x = -2, 6$, **j** $x = -0.30, -6.70$, **k** $x = 3.45, -1.45$, **l** $x = 8.53, 0.47$, **m** $x = -\frac{3}{2}, -4$, **n** $x = \frac{2}{3}, -5$, **o** $x = \frac{1}{2}, \frac{2}{3}$,

ANSWERS

p $x = 2.22, 0.45$, **q** $x = 4.53, -0.28$, **r** $x = 0.27, -0.37$ **2** **a** $x = 0, 5$, **b** $x = 0, 2$, **c** $x = 0, 4$, **d** $x = -4, 3$, **e** $x = 5, 6$, **f** $x = 2, 3$, **g** $x = \frac{1}{3}, -3$, **h** $x = \frac{5}{2}, 4$, **i** $x = -\frac{7}{4}, 2$, **j** $x = 6.61, -0.61$, **k** $x = 1.77, -6.77$, **l** $x = 1.27, 0.39$ **3** **a** $x = -3, -5$, **b** $x = 2, 7$, **c** $x = -5, 2$, **d** $x = -4, 7$, **e** $x = -\frac{9}{2}, 1$, **f** $x = \frac{4}{3}, 5$, **g** $x = 5.83, 0.17$, **h** $x = 11.20, 0.80$, **i** $x = 0.79, -3.79$, **j** $x = 2.89, -1.39$, **k** $x = 0.41, -4.91$, **l** $x = 1.55, 0.74$ **4** **a** $x = -8, 3$, **b** $x = -2, 4$, **c** $x = 3, 9$, **d** $x = \frac{3}{2}, 4$, **e** $x = -2, 8$, **f** $x = -12, 3$, **g** $x = 1.24, -3.24$, **h** $x = 5.65, 0.35$, **i** $x = -1.13, -3.54$, **j** $x = 9.11, -0.11$, **k** $x = -0.65, -3.85$, **l** $x = 0.39, -0.28$ **5** **a** $x = 3, 4$, **b** $x = -5, 8$, **c** $x = -7, 5$, **d** $x = 3, 6$, **e** $x = -2, 10$, **f** $x = -\frac{1}{2}, 5$, **g** $x = \frac{7 \pm \sqrt{37}}{2}$, **h** $x = \frac{1 \pm \sqrt{13}}{2}$, **i** $x = \frac{2 \pm \sqrt{6}}{2}$, **j** $x = 2 \pm \sqrt{3}$, **k** $x = 3 \pm \sqrt{11}$, **l** $x = \frac{4 \pm \sqrt{10}}{2}$

6 **a** $x = -\frac{7}{3}, 3$, **b** $x = \frac{3}{2}, -10$, **c** $x = \frac{3}{2}, \frac{1}{3}$, **d** $x = 0.88, -1.13$, **e** $x = 1.63, -1.23$, **f** $x = 0.52, -1.66$

7 **a** $x = 2$, $y = 4$ and $x = -2$, $y = 4$, **b** $x = 0$, $y = 0$ and $x = -3$, $y = 9$, **c** $x = 3$, $y = 9$ and $x = -1$, $y = 1$, **d** $x = 4$, $y = 16$ and $x = 5$, $y = 25$, **e** $x = 1$, $y = 2$ and $x = -1$, $y = 0$, **f** $x = 1$, $y = 4$ and $x = 5$, $y = 28$, **g** $x = 3$, $y = 6$, **h** $x = 3$, $y = 4$ and $x = -5$, $y = 4$, **i** $x = 4$, $y = 10$ and $x = -2$, $y = 4$, **j** $x = 3$, $y = 2$ and $x = 4$, $y = 4$, **k** $x = 1\frac{2}{3}$, $y = 8\frac{1}{3}$ and $x = -2$, $y = 12$, **l** $x = 2\frac{1}{2}$, $y = 12\frac{1}{2}$ and $x = 2$, $y = 8$ **8** **a** $x = -1, 1, -2, 2$, **b** $x = -1, 1, -3, 3$, **c** $x = -2, 2, -3, 3$, **d** $x = -1, 1$, **e** $x = -2, 2$, **f** $x = -3, 3$, **g** $x = -1, 1, -4, 4$, **h** $x = -3, 3$, **i** $x = -2, 2, -5, 5$, **j** $x = -5, 5$, **k** $x = -1, 1, -7, 7$, **l** $x = -2, 2, -4, 4$ **9** **a** $x = -1, 1$, **b** $x = -3, 3$, **c** $x = -\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}, -2, 2$

10 **a** $x = 0, 2$, **b** $x = 2, 3$, **c** $x = 0, 3$

Exercise 6.6

1 **a** 5, 6, **b** 7, 8, **c** 4, 6, **d** 7, 9, **e** 8, 11 **2** **a** 8, **b** 7 **3** −3, 5 **4** **a** 5, 8, **b** 2, 7, **c** 5, 6, **d** 8, 9, 10, 11 **5** 15 cm × 12 cm **6** 12 cm **7** **a** $x = 6$, **b** 24 cm^2 **8** 11 **9** **a** 2 s, 3 s, **b** 5 s, **c** No, the greatest height is 100 m when $t = 2.5$ s. **10** 2 m **11** $x = 4$ **12** 5 cm, 9 cm **13** **a** 3, **b** 4, **c** 20 **14** $x = 8$ **15** $\frac{2}{5}, \frac{5}{2}$ **16** **b** 7 cm × 4 cm **17** 17 cm × 6 cm **18** **a** $A = 100x - 2x^2$, **b** 40 m × 30 m, 60 m × 20 m

Chapter 6 Review

1 **a** yes, **b** yes, **c** no, **d** yes, **e** yes, **f** no **2** **a** $y = x^2 - 4$, **b** $y = 2x^2 + 3$ **3** **a** $y = x^2 + 3x - 2$, **b** $y = x^2 - 4x + 7$ **4** **a** $m = 0, 6$, **b** $x = 0, -8$, **c** $t = 0, -1$, **d** $n = 0, \frac{2}{3}$, **e** $p = 3, -5$, **f** $k = \frac{2}{3}, -\frac{5}{2}$ **5** **a** $p = 0, 4$, **b** $u = 0, -3$, **c** $n = -7, 7$, **d** $e = -3, 3$, **e** $x = -4, -6$, **f** $c = 4, -3$, **g** $y = 6$, **h** $a = -8, 4$, **i** $q = \frac{3}{2}, 7$, **j** $t = \frac{4}{5}, -5$

6 **a** $a = -4, 4$, **b** $t = 0, 5$, **c** $b = 5, -3$, **d** $y = 4, -2$, **e** $k = 2, 11$, **f** $w = \frac{1}{2}, -4$ **7** **a** $x = 4, 16$, **b** $x = -6 \pm 5\sqrt{2}$

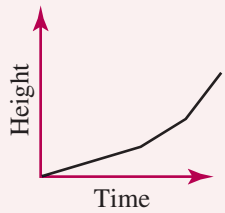
8 **a** $x = -3.83, 1.83$, **b** $x = 7.90, -1.90$, **c** $x = 0.56, -3.56$, **d** $x = 4.30, 0.70$ **9** **a** $x = -2, -3$, **b** $x = \frac{2}{3}, -5$, **c** $x = \frac{1 \pm \sqrt{21}}{2}$, **d** $x = \frac{9 \pm \sqrt{33}}{6}$ **10** **a** $x = 7, -5$, **b** $x = 0.44, -3.77$, **c** $x = 2, 10$, **d** $x = 0.80, -7.47$, **e** $x = 6, -5$, **f** $x = 4, 7$, **g** $x = \frac{3}{4}, -6$ **11** **a** 4, 5, **b** 6, 10, **c** 2, 9 **12** **a** 12 cm × 7 cm, **b** 38 cm **13** $x = 8$ **14** **a** 6 s, **b** 2 s, 4 s, **c** 3 s

7 Graphs in the number plane

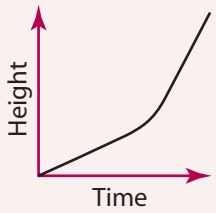
Exercise 7.1

1 **a** R, **b** P, **c** P, **d** Q **2** **a** B, **b** C, **c** A, E, **d** D, E **3** **a** **i** Tony, **ii** Pete, **iii** Quentin, **iv** Robin, **b** **i** Quentin, Robin, **ii** Pete, Steve **4** **a** A, D, **b** B, C **c** D, **d** B **5** B **6** C **7** B **8** D **9** A **10** A **11** A **12** **a** B, **b** C, **c** A **13** **a** D, **b** B, **c** C, **d** A

14 a



b



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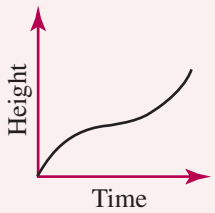
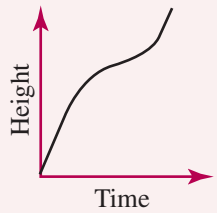
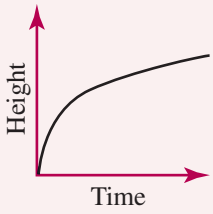


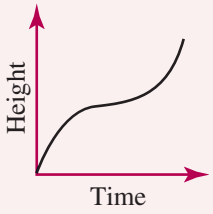
d



15

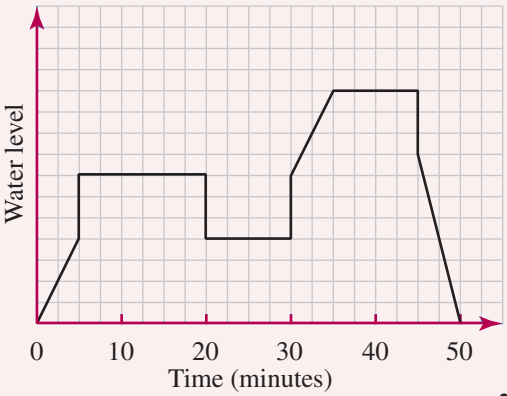


16

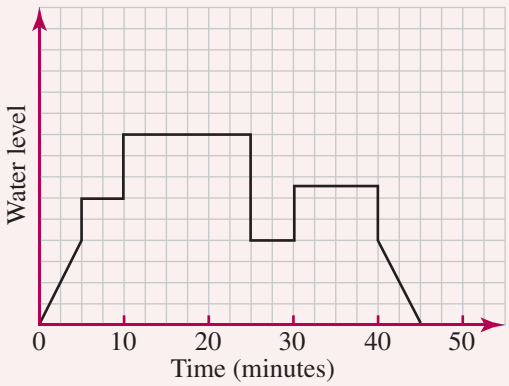


18 a She probably opened a new account, since the opening balance is $0.
b Tuesday, Wednesday, Friday,
c Friday,
d Thursday,
e Sunday

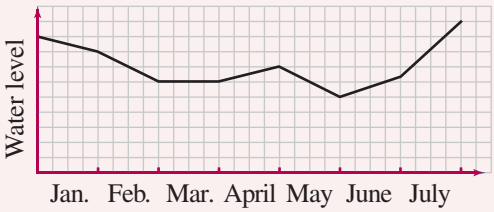
19 a



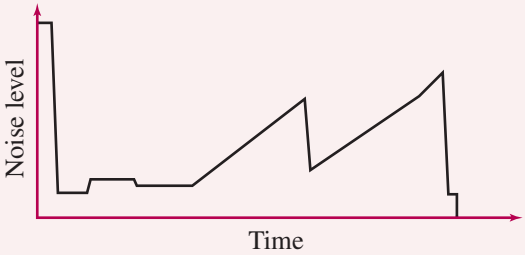
b



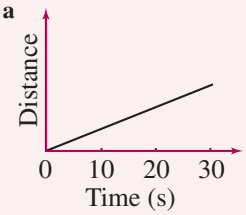
20



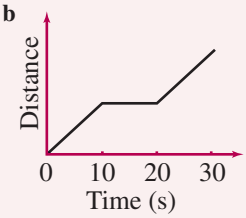
21



22 a

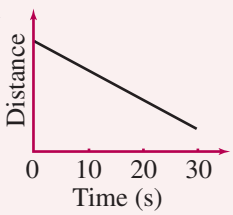


b



c

d



Exercise 7.2

1 a 40 km, **b** Her speed increased, the line became steeper. **c** noon, **d** $1\frac{1}{2}$ h, **e** 100 km, **f** $2\frac{1}{2}$ h, **g** 200 km

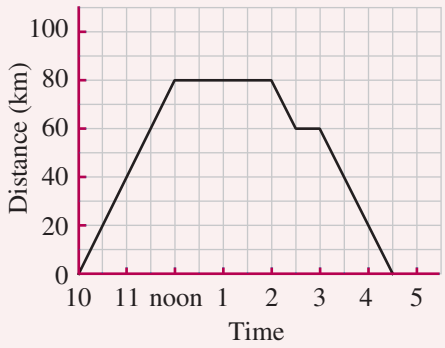
2 a 9 am, **b** 80 km, **c i** 100 km/h, **ii** 60 km/h, **d** noon, **e** 100 km, **f** 25 km **3 a** 16 km, **b** 6 km, **c** 10 am–noon, **d** 18 km, **e** 14 km, **f** 3.30 pm, **g** 32 km **4 a** 150 km, **b** 1.30 pm–2 pm, **c** 10.30 am,

d 8.30 am, 9.45 am, 12.30 pm, **e** 9am–9.30 am, 10.30 am–11.30 am, **f** $5\frac{1}{2}$ h, **g** 165 km

5 a 6 am, **b** 6.30 am, 9.30 am, **c** 30 km, **d** Steve, **e** 30 km, **f** 180 km, **g** Steve — 40 km/h, Perry — 80 km/h
6 a 200 km, **b** 11 am, 3.30 pm, **c** 150 km, **d** Sourav, **e** Irena, by 100 km, **f** 3.30 pm–5.30 pm, **g** 54.5 km/h

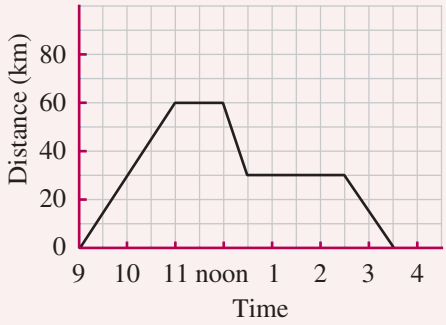
ANSWERS

7 a



b 40 km/h, **c** 3 pm, **d** 4.30 pm, **e** 11.15 am, 3.15 pm

8 a

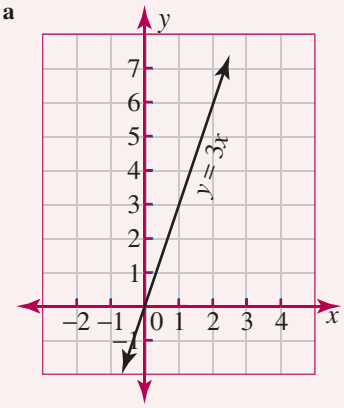


b 2 h, **c** 12.30 pm, **d** 30 km/h, **e** 120 km

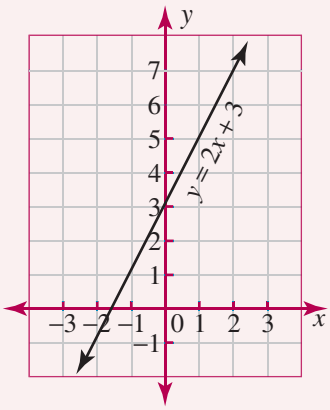
22 a

Exercise 7.3

1 a



b



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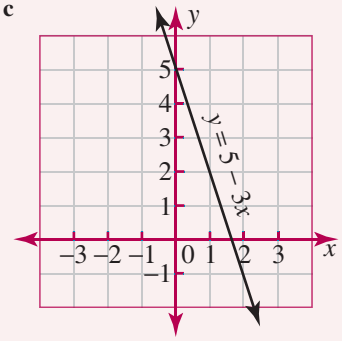

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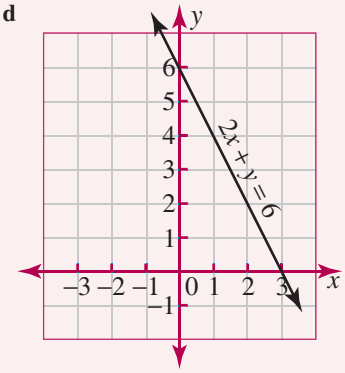
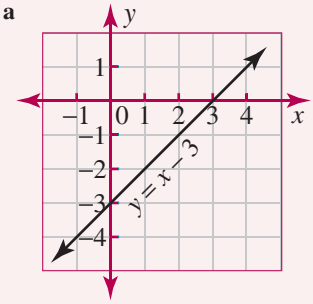


2 a



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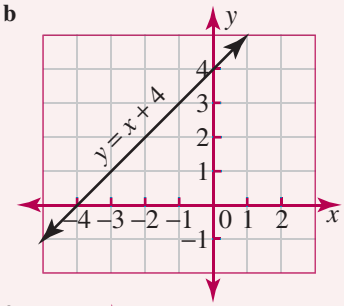

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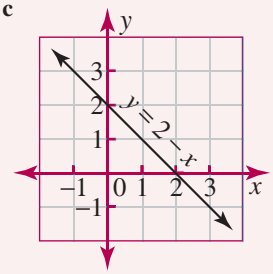

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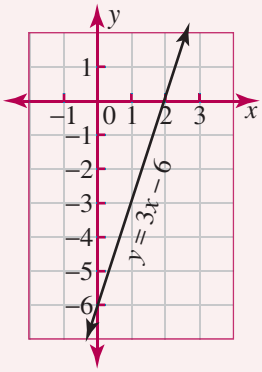

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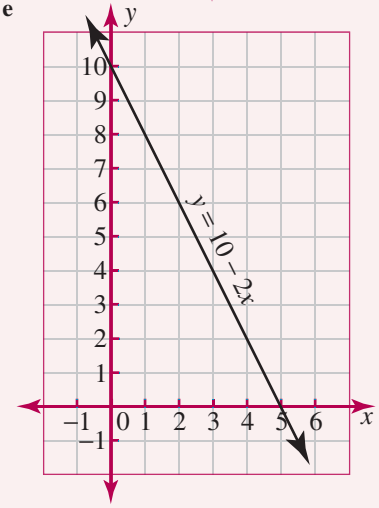

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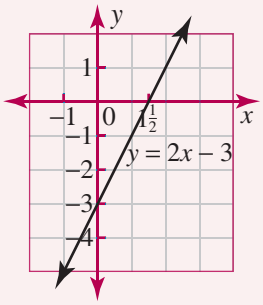

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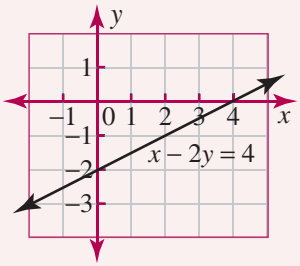

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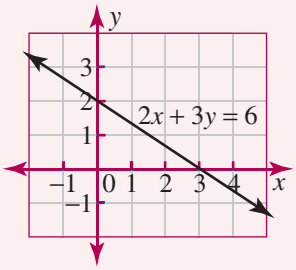

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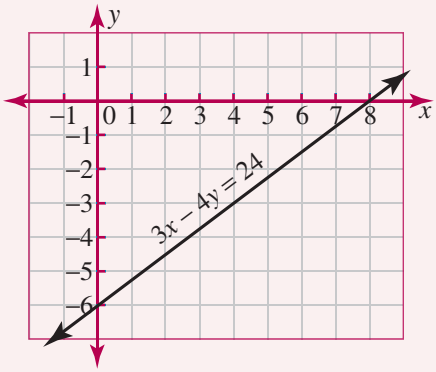

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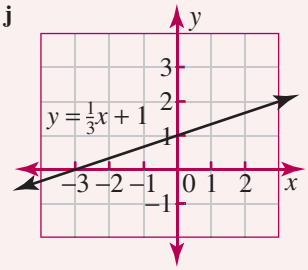

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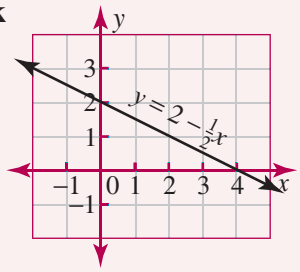
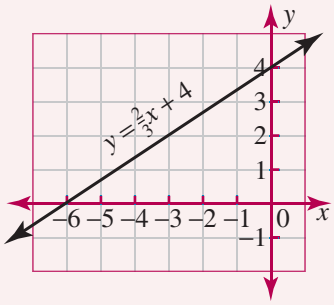


l

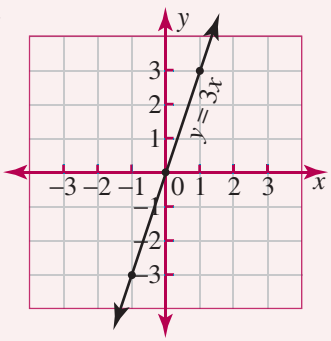


3 **a** $m = 1, b = 7$, **b** $m = 2, b = -5$, **c** $m = 3, b = 4$, **d** $m = -1, b = -6$, **e** $m = 4, b = 0$, **f** $m = -5, b = 8$, **g** $m = \frac{1}{2}, b = 4$, **h** $m = \frac{5}{6}, b = 0$

4 **a** $y = 2x + 3$, **b** $y = -x + 4$, **c** $y = 3x - 2$, **d** $y = -\frac{1}{2}x - 6$, **e** $y = \frac{2}{3}x$

5 **a** $y = 2x + \frac{3}{2}$, $m = 2, b = \frac{3}{2}$, **b** $y = \frac{1}{3}x - \frac{1}{6}$, $m = \frac{1}{3}, b = -\frac{1}{6}$, **c** $y = -\frac{3}{4}x - 3$, $m = -\frac{3}{4}, b = -3$, **d** $y = -\frac{5}{3}x + \frac{11}{3}$, $m = -\frac{5}{3}, b = \frac{11}{3}$

6 **a**



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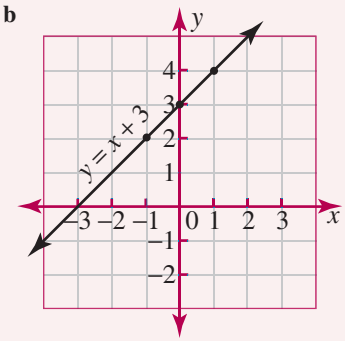
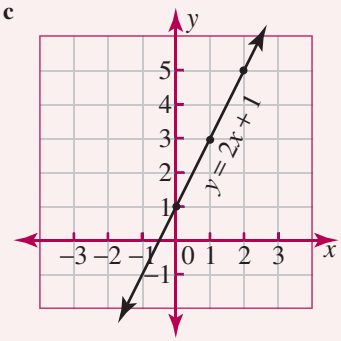
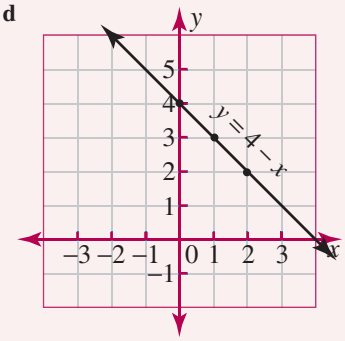


c



ANSWERS

d



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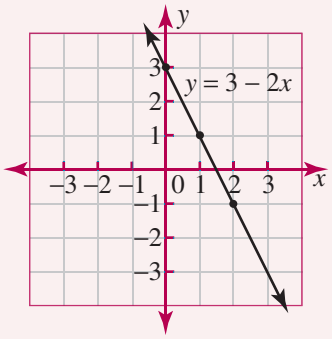

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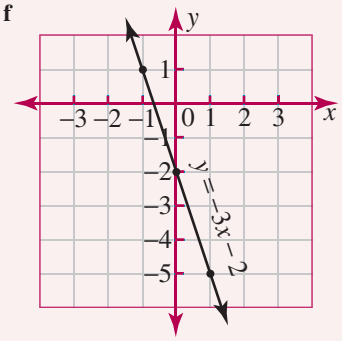

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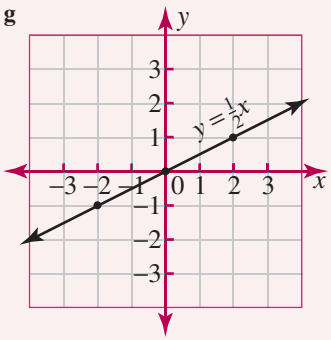

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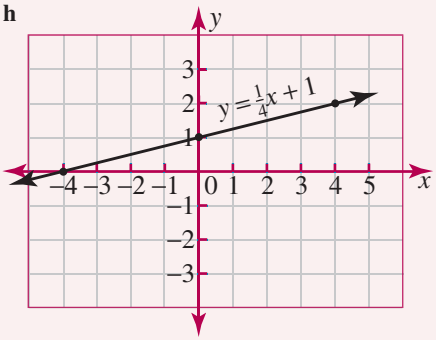

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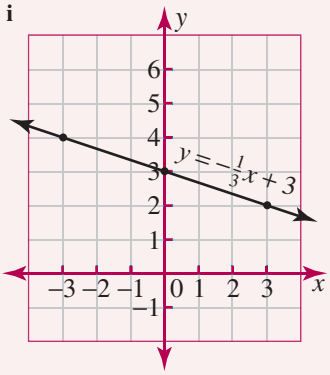

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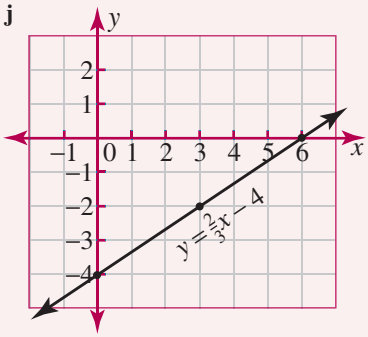

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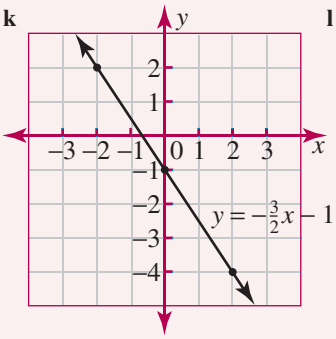

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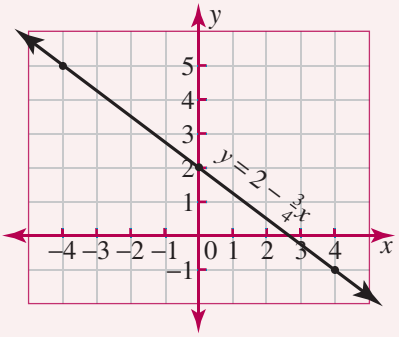


7 **a** $y = -3x - 4$, **b** $y = 3x + 4$, **c** $y = 4 - 3x$, **d** $y = 3x - 4$ **8** **a** $y = 3x$, **b** $y = -\frac{1}{3}x$, **c** $y = -3x$, **d** $y = \frac{1}{3}x$

9 **a** $y = x + 5$, **b** $y = 2x + 6$, **c** $y = -3x + 12$, **d** $y = \frac{1}{2}x - 2$, **e** $y = \frac{5}{3}x - 5$, **f** $y = -\frac{2}{3}x - 4$ **10** **a** $x = 2$, **b** $y = 1$, **c** $y = -2$, **d** $x = -1$, **e** $y = 4$, **f** $x = -3$, **g** $x = 3$, **h** $y = -4$ **11** **a** $y = 5$, **b** $x = 2$, **c** $x = -3$, **d** $y = -5$ **12** **a** yes, **b** no, **c** no, **d** yes, **e** no, **f** yes **13** **a** no, **b** yes, **c** yes, **d** no, **e** yes, **f** no, **g** yes, **h** yes **14** **a** no, **b** yes, **c** yes, **d** no, **e** no, **f** yes, **g** yes, **h** no **15** **a** $c = -6$, **b** $g = 7$, **c** $m = 11$, **d** $z = -8$ **16** **a** $y = x + 7$, **b** $y = 5x$, **c** $y = 2x + 3$, **d** $y = 4x - 1$, **e** $y = -x + 8$, **f** $y = 9x - 5$, **g** $y = -2x + 7$, **h** $y = \frac{1}{2}x - 2$ **17** **a** **i** $y = 9$, **ii** $y = 11$, **iii** $y = 0$, **b** **i** $x = 9$, **ii** $x = 1, 6$, **iii** $x = 2$, **c** $y = 2x + 3$ for $0 \leqslant x \leqslant 4$, $y = -x + 11$ for $4 < x \leqslant 11$ **18** **a** **i** $y = 8$, **ii** $y = 5$, **iii** $y = 4$, **b** **i** $t = 7$, **ii** $y = 5\frac{1}{2}, 6\frac{1}{2}, 8$, **iii** $3 \leqslant t \leqslant 5$, $t = 6, 9$

Exercise 7.4

1 a

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9

c no, **d** $x = 0$, **e** (0, 0), **f** no, **g** $y = 0$, **h** concave upward

2 a 1, **b** 2 **3 a** 0.2, **b** 2.6, **c** 4.8, **d** 7.3, **e** 0.4, **f** 1.7, **g** 3.2, **h** 4.4

4 a 1.4, **b** 2.2, **c** 2.6, **d** 2.8, **e** 0.9, **f** 1.8, **g** 2.5, **h** 2.7

5 a $x = \pm 1.7$, **b** $x = \pm 2.8$, **c** $x = \pm 1.3$, **d** $x = \pm 2.4$

6 a i

x	−2	−1.5	−1	−0.5	0	0.5	1	1.5	2
y	4	2.25	1	0.25	0	0.25	1	2.25	4

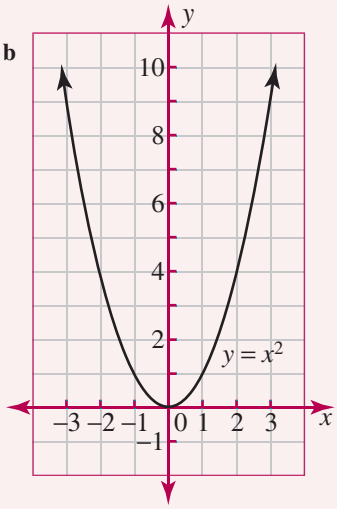
ii

x	−2	−1.5	−1	−0.5	0	0.5	1	1.5	2
y	8	4.5	2	0.5	0	0.5	2	4.5	8

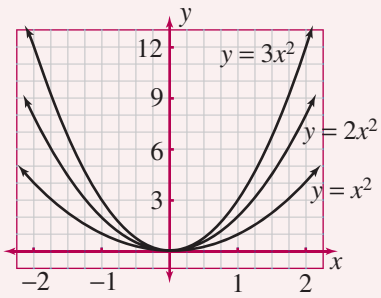
iii

x	−2	−1.5	−1	−0.5	0	0.5	1	1.5	2
y	12	6.75	3	0.75	0	0.75	3	6.75	12

b



b



c The curve becomes steeper as the value of a decreases.

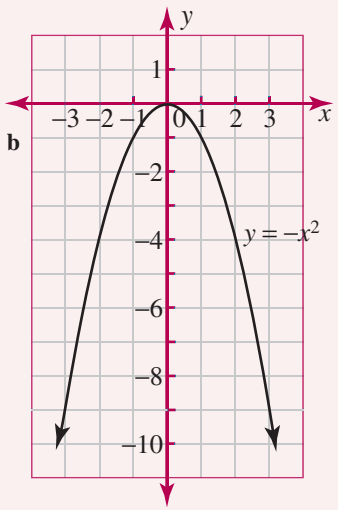
7 a B, **b** C, **c** A

8 a

x	−3	−2	−1	0	1	2	3
y	−9	−4	−1	0	1	4	9

c no, **d** $x = 0$, **e** (0, 0), **f** no, **g** $y = 0$, **h** concave downward

b



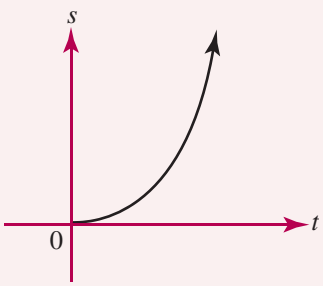
9 a If $a > 0$, the parabola is concave upward.
b If $a < 0$, the parabola is concave downward.

10 a B, **b** C, **c** A, **d** D

11 a $y = 2x^2$, **b** $y = 5x^2$, **c** $y = \frac{1}{2}x^2$, **d** $y = -2x^2$, **e** $y = -3x^2$, **f** $y = -\frac{1}{3}x^2$

12 a $t \geqslant 0$, time cannot be negative.

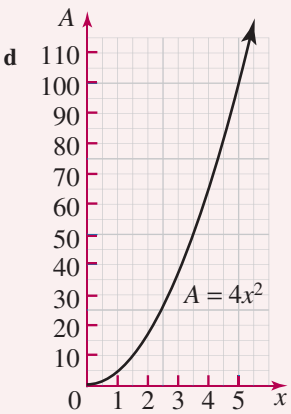
b



c i 5 m, **ii** 20 m, **iii** 245 m,
d i 3 s, **ii** 14 s, **iii** 20 s, **e** 25 m

13 a $4x$, **b** $A = 4x^2$, **c** $x > 0$, the dimensionscannot be negativeor zero.
e i 36 cm², **ii** 64 cm², **iii** 100 cm²,
g 14 cm × 3.5 cm

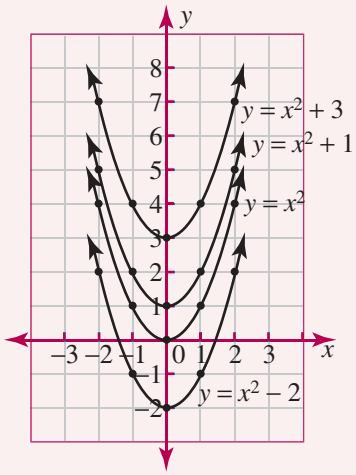
d



ANSWERS

Exercise 7.5

1



a

x	−2	−1	0	1	2
y	4	1	0	1	4

b

x	−2	−1	0	1	2
y	5	2	1	2	5

c

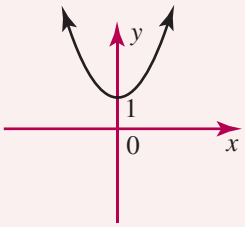
x	−2	−1	0	1	2
y	7	4	3	4	7

d

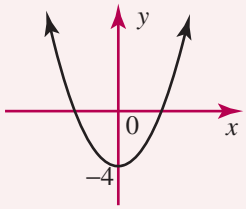
x	−2	−1	0	1	2
y	2	−1	−2	−1	2

2 The graphs of $y = x^2 + c$ and $y = x^2 - c$ $(c > 0)$ are obtained by translating the parabola $y = x^2$ upward or downward by c units respectively.
3 **a** translate $y = x^2$ upward by 4 units, **b** translate $y = x^2$ downward by 5 units, **c** translate $y = x^2$ upward by 7 units, **d** translate $y = x^2$ downward by 12 units

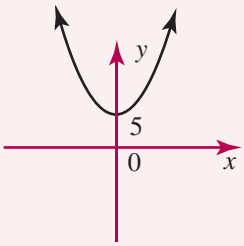
4 **a**



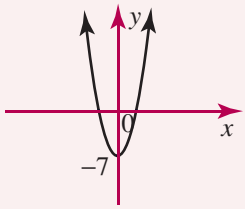
b



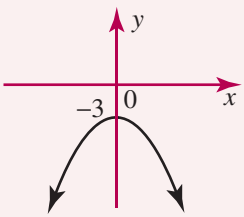
c



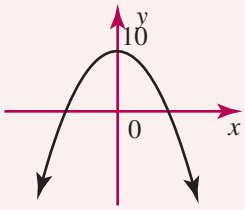
d



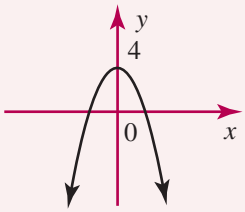
e



f



g



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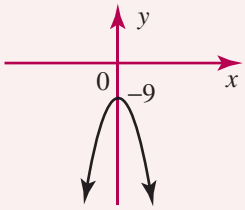
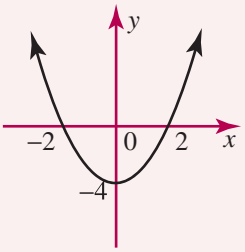


5 $(0, -8)$ **6** **a** $y = 3 - x^2$, **b** $y = x^2 + 3$, **c** $y = -x^2 - 3$, **d** $y = x^2 - 3$ **7** **a** $y = x^2 + 8$, **b** $y = x^2$, **c** $y = x^2 - 3$

8 **a**



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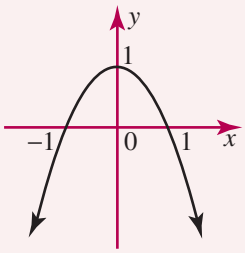

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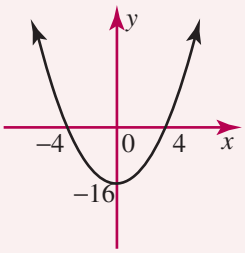

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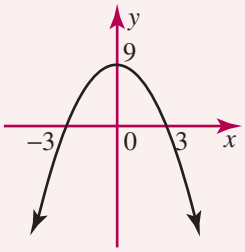
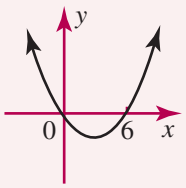


9 $y = 9 - x^2$ **10** **a** $y = x^2 + 5$, **b** $y = x^2 - 4$, **c** $y = x^2 - 7$, **d** $y = 14 - x^2$, **e** $y = -x^2 - 5$, **f** $y = 2 - x^2$
11 **a** $y = 2x^2 - 3$, **b** $y = 7 - 4x^2$ **12** $y = x^2 + 3$ **13** $y = 3 - x^2$

Exercise 7.6

1 a



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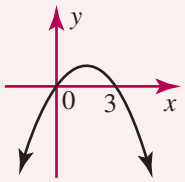

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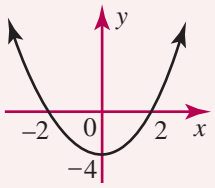

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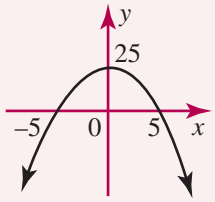

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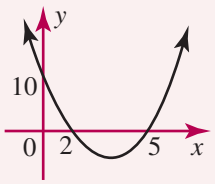

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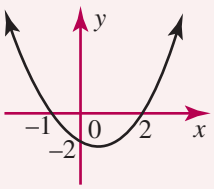

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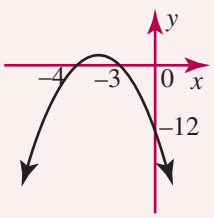

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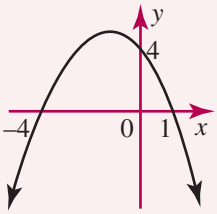

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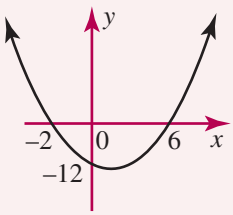
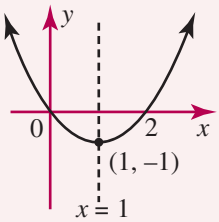
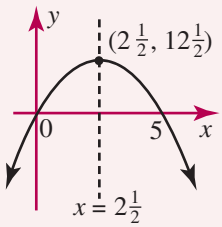


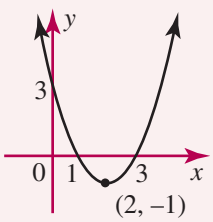
2 a i $x = 0, 2,$
ii $y = 0,$
iii $x = 1$, $V(1, -1)$,
iv



b i $x = 0, 5$
ii $y = 0$
iii $x = 2\frac{1}{2}$, $V(2\frac{1}{2}, 12\frac{1}{2})$
iv



3 a



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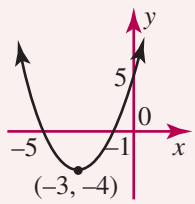

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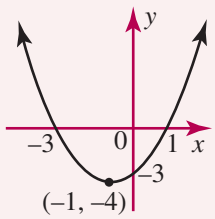

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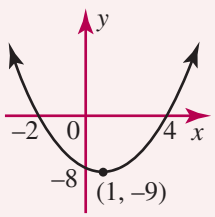

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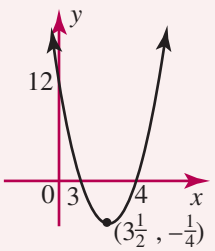

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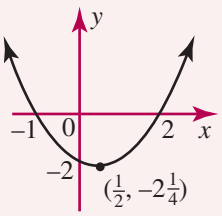

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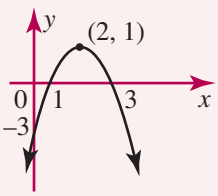

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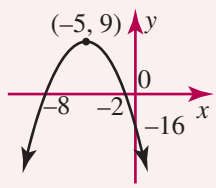

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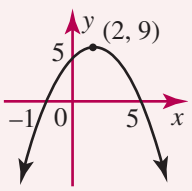

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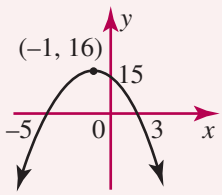

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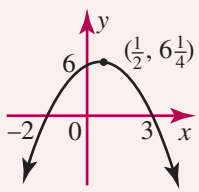

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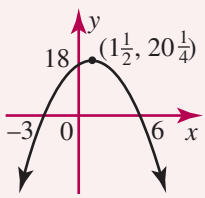
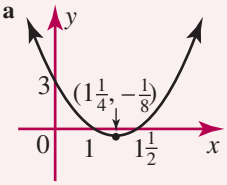


ANSWERS

5 a



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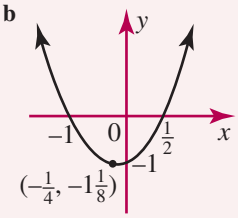

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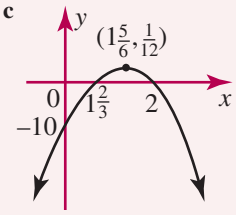

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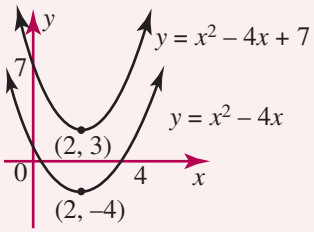

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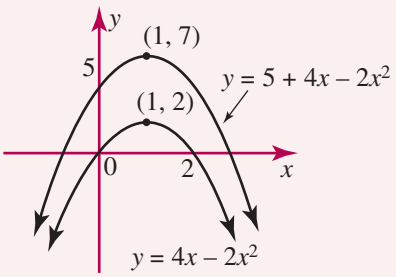

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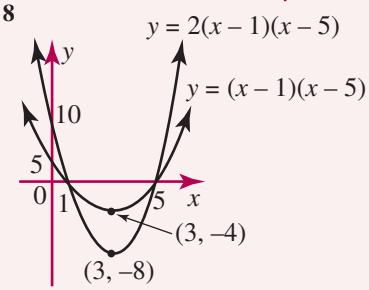
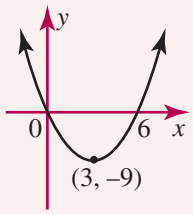


9 a $x = 5 \pm \sqrt{3}$, **b** $x = \frac{9 \pm \sqrt{93}}{2}$, **c** $x = -3 \pm \sqrt{5}$ **10 a** $y = (x-2)(x-7)$, **b** $y = 2(x+3)(x-1)$,
c $y = \frac{1}{2}(x+2)(x-6)$, **d** $y = -(x-4)(x-6)$, **e** $y = -3(x+3)(x-2)$, **f** $y = -\frac{2}{3}(x+4)(x+1)$ **11** $y = 2x^2 - x + 4$

Exercise 7.7

1 a $x = 3$, **b** $(3, -9)$, **c** $x = 0, 6$

d



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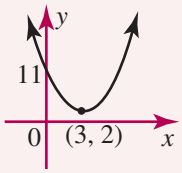

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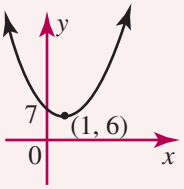

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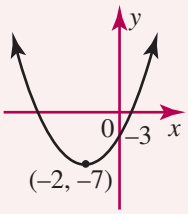

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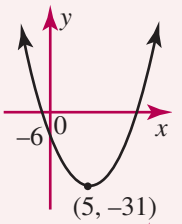

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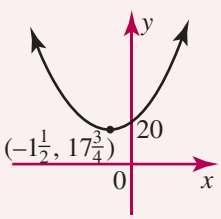

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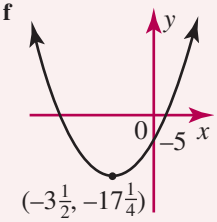

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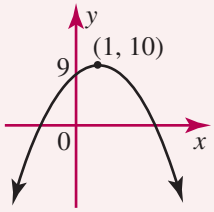

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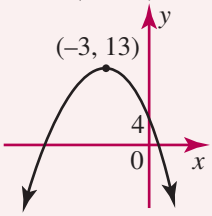

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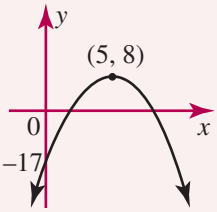

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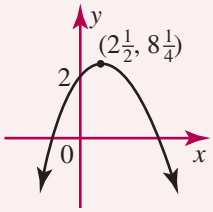

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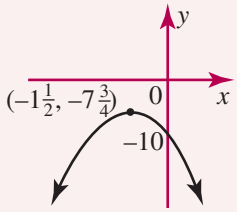

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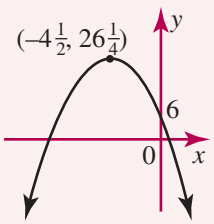
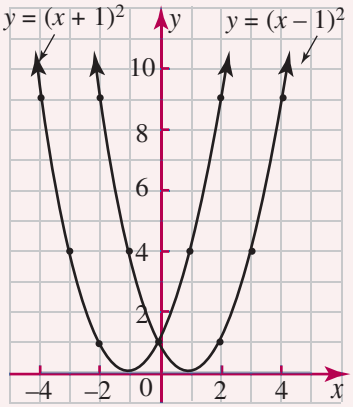


3 a $(-2, -1)$, **b** $(1, -5)$, **c** $(4, 45)$, **d** $(-2, 4)$, **e** $(-2\frac{1}{2}, -17\frac{1}{2})$, **f** $(2, 32)$

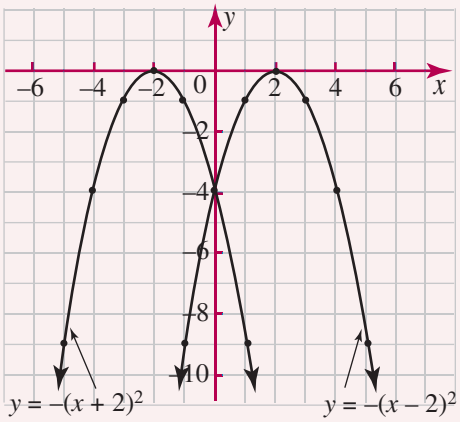
4 a min. $= 5\frac{7}{8}$, **b** min. $= -4\frac{1}{12}$, **c** max. $= 16\frac{1}{8}$, **d** min. $= -23\frac{1}{4}$, **e** max. $= -4\frac{7}{16}$,
f max. $= 5\frac{1}{3}$ **5** -5 **6 a** $y = x^2 + 4x - 10$, **b** $y = x^2 + 4x + 10$, **c** $y = x^2 - 4x + 10$,
d $y = x^2 - 4x - 10$ **7** $k = -12$ **8 a** $b = -18$, $c = 11$, **b** 59
9 max. value $= 15$, when $x = 4$

Exercise 7.8

1 a

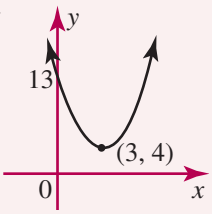


b



2 a $y = (x - 1)^2$ – shift $y = x^2$ one unit to the right; $y = (x + 1)^2$ – shift $y = x^2$ one unit to the left.
b $y = -(x - 2)^2$ – shift $y = -x^2$ two units to the right; $y = -(x + 2)^2$ – shift $y = -x^2$ two units to the left.

3 a



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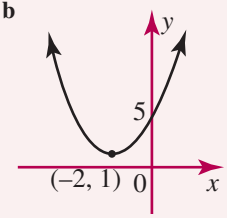

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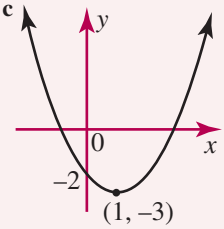
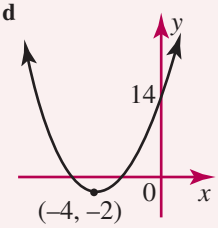


d



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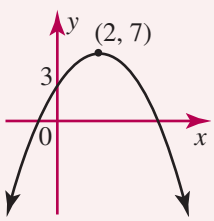

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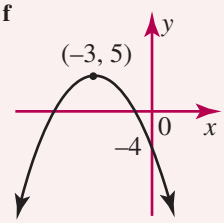

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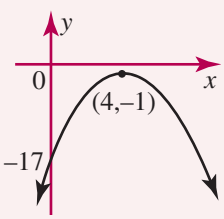

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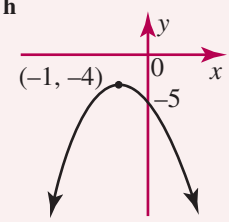
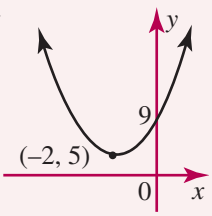


4 a



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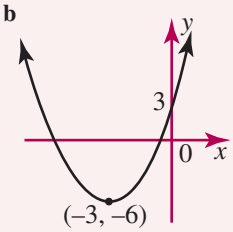

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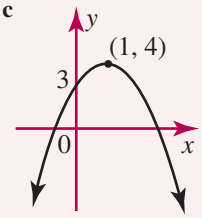

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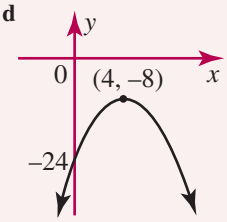

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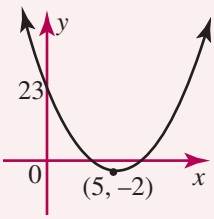

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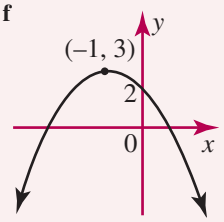

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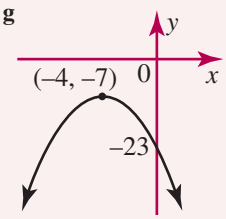

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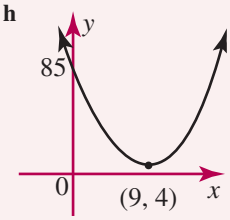
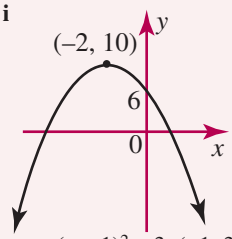


ANSWERS

i



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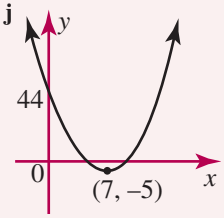

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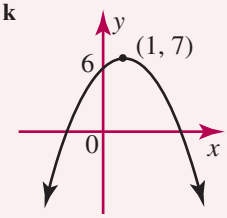

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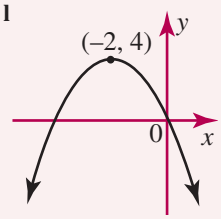
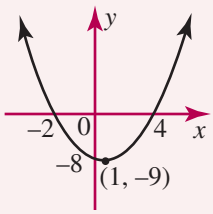


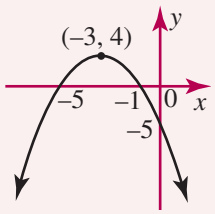
5 **a** $y = (x + 1)^2 + 3$, $(-1, 3)$, **b** $y = (x + 3)^2 + 4$, $(-3, 4)$, **c** $y = (x - 2)^2 + 5$, $(2, 5)$, **d** $y = (x - 1)^2 + 4$, $(1, 4)$,
e $y = (x + 3)^2 - 1$, $(-3, -1)$, **f** $y = (x - 4)^2 - 5$, $(4, -5)$, **g** $y = (x + 2)^2 - 7$, $(-2, -7)$, **h** $y = (x - 4)^2 + 5$, $(4, 5)$,
i $y = (x - 2)^2 - 9$, $(2, -9)$, **j** $y = (x + 3)^2 + 1$, $(-3, 1)$, **k** $y = (x - 5)^2 - 12$, $(5, -12)$, **l** $y = (x - 4)^2 - 20$, $(4, -20)$,
m $y = (x + 1)^2 - 7$, $(-1, -7)$, **n** $y = (x + 5)^2 + 7$, $(-5, 7)$, **o** $y = (x - 6)^2 + 3$, $(6, 3)$

6 **a** $y = (x + 1\frac{1}{2})^2 + 1\frac{3}{4}$, $V(-1\frac{1}{2}, 1\frac{3}{4})$, **b** $y = (x - 2\frac{1}{2})^2 + 2\frac{3}{4}$, $V(2\frac{1}{2}, 2\frac{3}{4})$, **c** $y = (x + \frac{1}{2})^2 - 5\frac{1}{4}$, $V(-\frac{1}{2}, -5\frac{1}{4})$

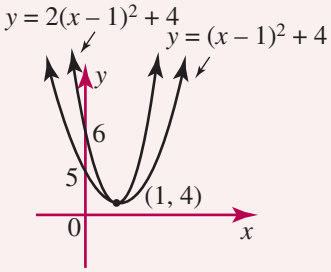
7 **a**



b



8 **a**



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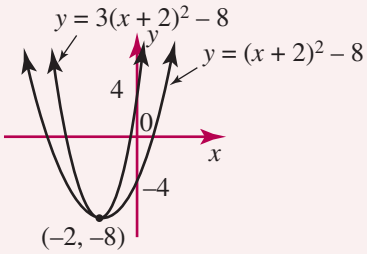
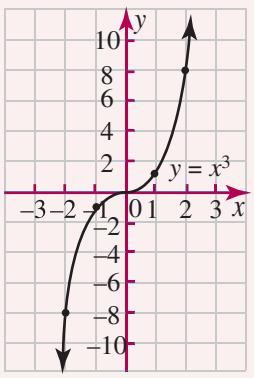


9 **a** $y = (x - 4)^2 + 7$, **b** $y = (x + 2)^2 - 1$, **c** $y = -(x - 1)^2 + 6$,
d $y = -(x + 5)^2 + 4$, **e** $y = -(x - 3)^2 - 10$, **f** $y = (x + 4)^2 + 8$

10 **a** $y = 2(x - 3)^2 + 6$, **b** $y = 3(x + 1)^2 - 10$, **c** $y = -2(x - 4)^2 + 9$,
d $y = -4(x + 2)^2 - 5$, **e** $y = -3(x + 3)^2 + 35$, **f** $y = 4(x - 6)^2 - 11$

Exercise 7.9

1 **a**



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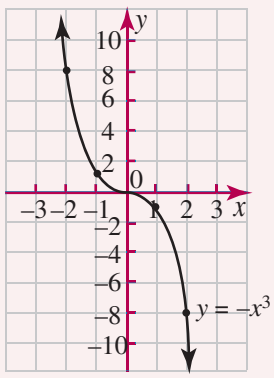
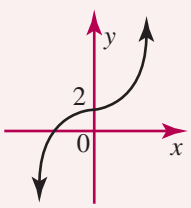
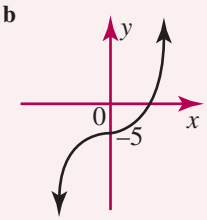


2 **a** translate up 1 unit
b translate down 2 units
c translate up 4 units
d translate down 7 units

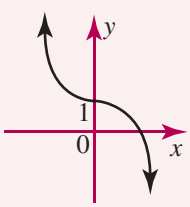
3 **a**



b



c



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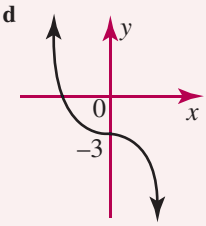
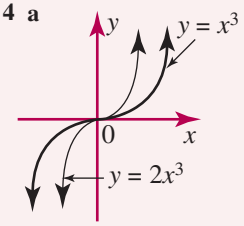
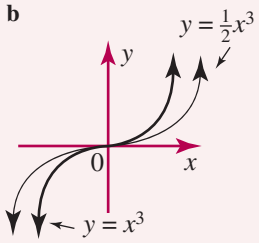


4 a



b



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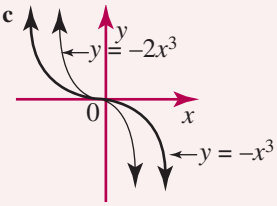
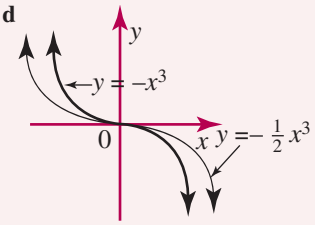


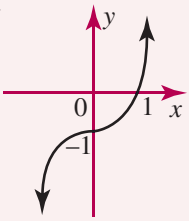
d



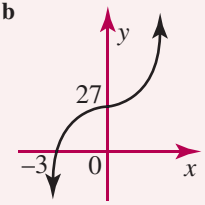
5 **a** (0, 0), **b** (0, 6), **c** (0, 3), **d** (0, −5) **6** **a** $y = x^3 - 2$, **b** $y = -x^3 + 2$, **c** $y = x^3 + 2$, **d** $y = -x^3 - 2$

7 **a** $y = x^3 + 7$, **b** $y = x^3$, **c** $y = x^3 - 2$

8 a



b



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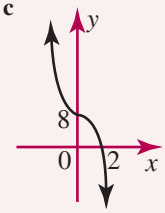

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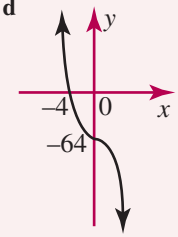


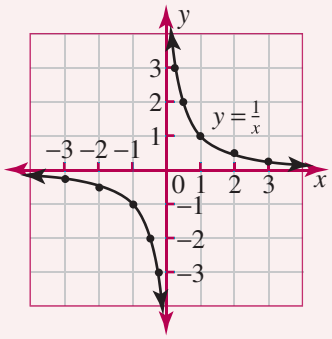
9 $y = -x^3 + 5$ **10** **a** $y = x^3$, **b** $y = \frac{3}{2}x^3$, **c** $y = -3x^3$ **11** **a** $y = x^3 + 6$, **b** $y = -x^3 + 4$, **c** $y = -\frac{1}{2}x^3 - 2$

Exercise 7.10

1 a

x	−3	−2	−1	$-\frac{1}{2}$	$-\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{1}{2}$	1	2	3
y	$-\frac{1}{3}$	$-\frac{1}{2}$	−1	−2	−3	–	3	2	1	$\frac{1}{2}$	$\frac{1}{3}$

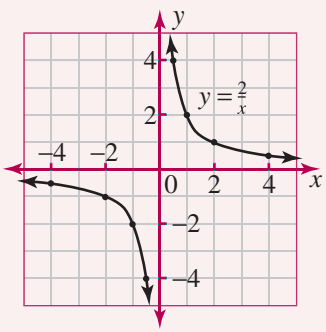
b



c 1st, 3rd, **d** **i** the y-values decrease, **ii** the y-values increase, **e** **i** y is positive, **ii** y is negative, **f** no, **g** asymptotes, **h** $y = x$, $y = -x$

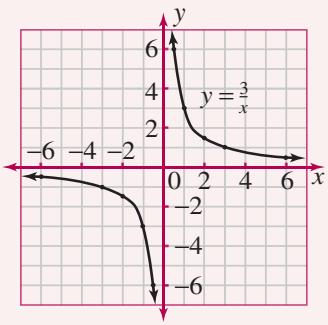
2 a

x	−4	−2	−1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	4
y	$-\frac{1}{2}$	−1	−2	−4	–	4	2	1	$\frac{1}{2}$



b

x	−6	−3	−2	−1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	3	6
y	$-\frac{1}{2}$	−1	$-1\frac{1}{2}$	−3	−6	–	6	3	$1\frac{1}{2}$	1	$\frac{1}{2}$

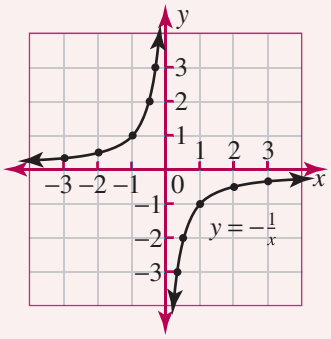


ANSWERS

3 a

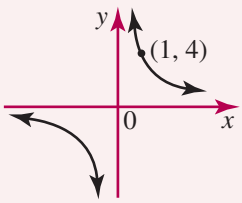
x	-3	-2	-1	$-\frac{1}{2}$	$-\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{1}{2}$	1	2	3
y	$\frac{1}{3}$	$\frac{1}{2}$	1	2	3	–	-3	-2	-1	$-\frac{1}{2}$	$-\frac{1}{3}$

b

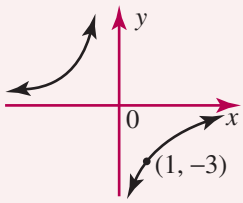


c 2nd, 4th, **d i** y is negative, **ii** y is positive, **e** yes, the x, y axes, **f** $y = x$, $y = -x$

4 a



b



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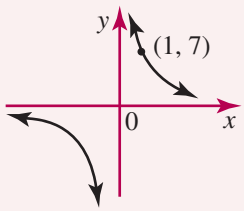
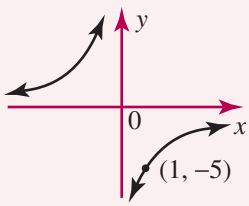


d



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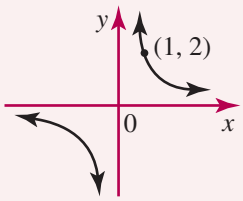

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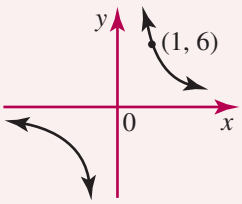

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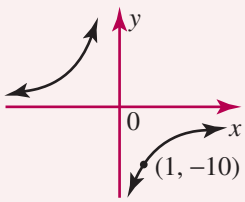
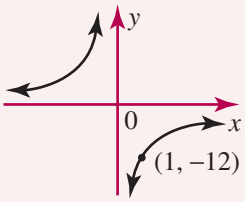


h

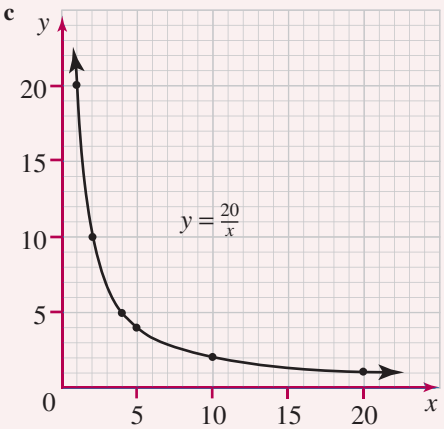


5 a $p = 6$, **b** $a = -4$, **c** $k = -1$, **d** $u = \frac{3}{2}$ **6 a** 4 units, **b** 15 units **7 a** $xy = 7$, **b** $xy = 10$, **c** $xy = -6$, **d** $xy = -20$, **e** $xy = 3$, **f** $xy = -8$ **8 a** $\frac{20}{x}$ cm,

b

x	1	2	4	5	10	20
y	20	10	5	4	2	1

c

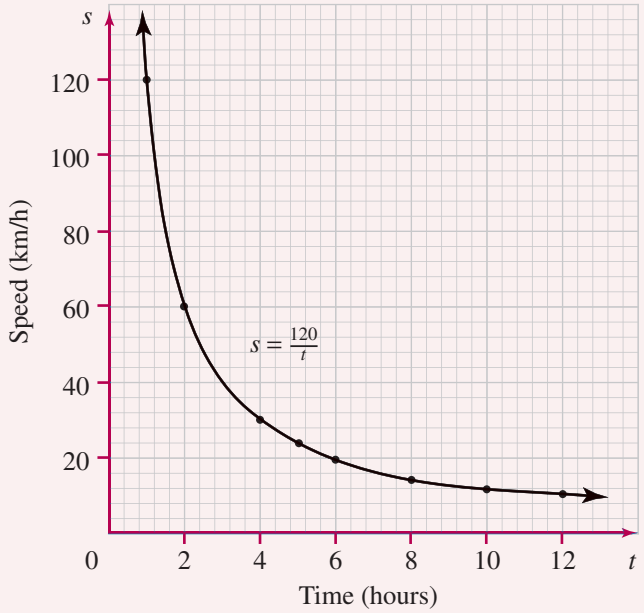


d Hyperbola ($x > 0$)

9 a

t	1	2	4	5	6	8	10	12
s	120	60	30	24	20	15	12	10

b



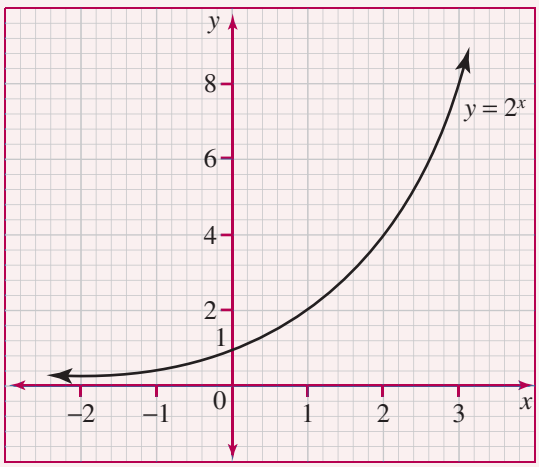
c 40 km/h

Exercise 7.11

1 a

x	−2	−1	0	1	2	3
y	0.25	0.5	1	2	4	8

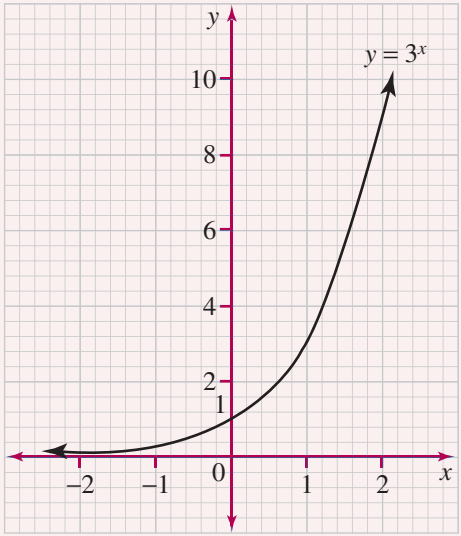
b



c **i** the y-values increase, **ii** the y-values decrease, **d** no, **e** asymptote, **f** the y-values are positive, **g** (0, 1), **h** no

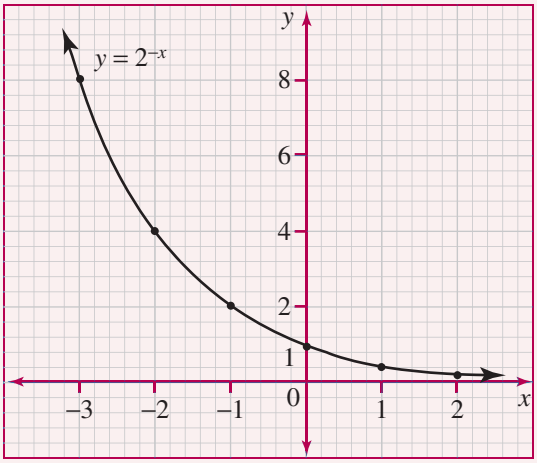
2

x	−2	−1.5	−1	−0.5	0	0.5	1	1.5	2
y	0.1	0.2	0.3	0.6	1	1.7	3	5.2	9



3

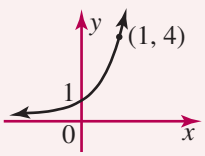
x	−3	−2	−1	0	1	2
y	8	4	2	1	0.5	0.25



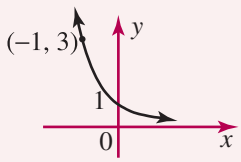
c **i** the y-values decrease, **ii** the y-values increase, **d** no, **e** the y-values are positive, **f** (0, 1), **g** no

4 Yes, $a° = 1$ for all values of a, where $a > 0$.

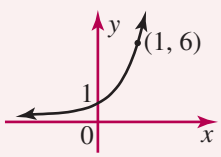
5 **a**



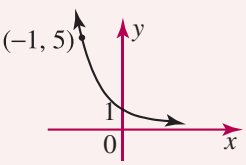
b



c

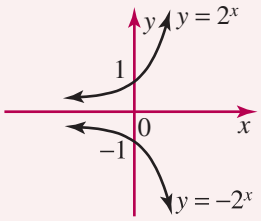


d

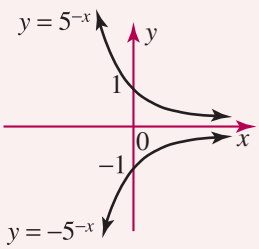


6 **a** $m = 16$, **b** $c = 3$, **c** $w = \frac{1}{2}$, **d** $h = -2$ **7** 63 units **8** **a** $y = 7^x$, **b** $y = 3^x$, **c** $y = 6^{-x}$

9 **a** Reflect the graph of $y = 2^x$ in the x-axis. **b**



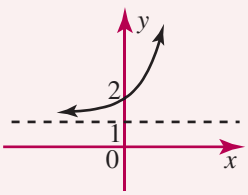
10



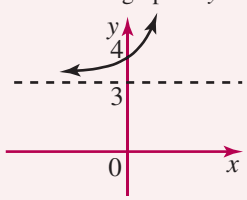
11 No, $6^x = (2 \times 3)^x = 2^x \times 3^x$

12 Reflect the graph of $y = 7^x$ in the y-axis.

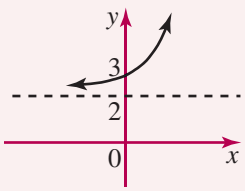
13 **a**



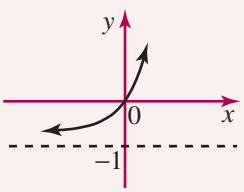
b



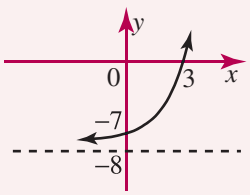
c



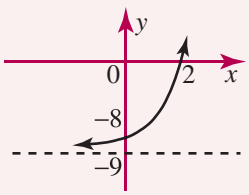
d

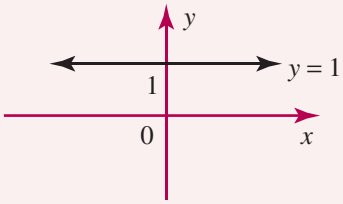


e



f

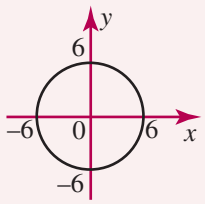


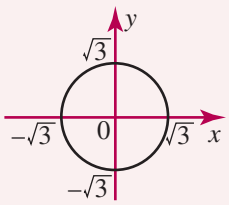
14 

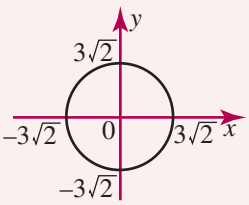
The graph of $y = 1^x$ is the graph of $y = 1$.

Exercise 7.12

1 **a** $x^2 + y^2 = 9$, **b** $x^2 + y^2 = 49$, **c** $x^2 + y^2 = 1$ **2** **a** $x^2 + y^2 = 4$, **b** $x^2 + y^2 = 25$, **c** $x^2 + y^2 = 16$, **d** $x^2 + y^2 = 3$, **e** $x^2 + y^2 = 7$, **f** $x^2 + y^2 = 10$, **g** $x^2 + y^2 = 8$, **h** $x^2 + y^2 = 48$, **i** $x^2 + y^2 = 63$, **j** $x^2 + y^2 = 1.44$, **k** $x^2 + y^2 = 3.61$, **l** $x^2 + y^2 = 6.25$ **3** **a** 3, **b** 7, **c** 11, **d** $\sqrt{5}$, **e** $\sqrt{14}$, **f** $\sqrt{22}$, **g** $2\sqrt{3}$, **h** $3\sqrt{3}$, **i** $3\sqrt{5}$, **j** 1.3, **k** 1.7, **l** 3.5

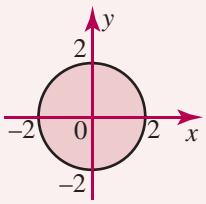
4 **a** 

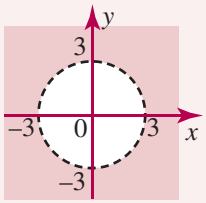
b 

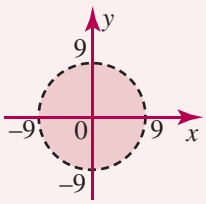
c 

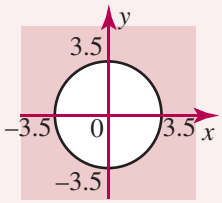
5 **a** $4x^2 + 4y^2 = 9$, **b** $16x^2 + 16y^2 = 121$, **c** $25x^2 + 25y^2 = 289$ **6** **a** $1\frac{1}{2}$, **b** $\frac{2}{3}$, **c** $2\frac{1}{2}$, **d** $\frac{4}{7}$, **e** $\frac{3}{4}$, **f** $2\frac{1}{5}$

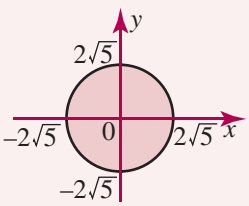
7 **a** (6, 8), (6, –8), **b** (6, –8), (–6, –8), **c** $(7, \sqrt{51})$, $(7, -\sqrt{51})$, **d** $(5\sqrt{3}, -5)$, $(-5\sqrt{3}, -5)$ **8** **a** $x^2 + y^2 = 25$, **b** $x^2 + y^2 = 34$, **c** $x^2 + y^2 = 17$ **9** **a** no, **b** yes, **c** no, **d** yes, **e** yes, **f** no, **g** yes, **h** no, **i** no **10** **a** inside, **b** outside, **c** on circle, **d** inside, **e** outside, **f** inside, **g** on circle, **h** outside

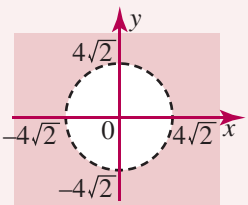
11 **a** 

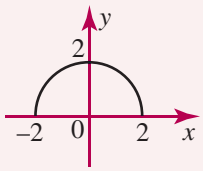
b 

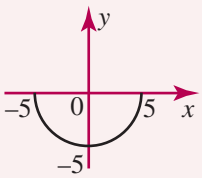
c 

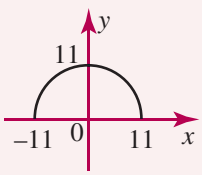
d 

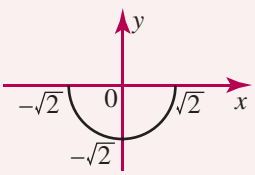
e 

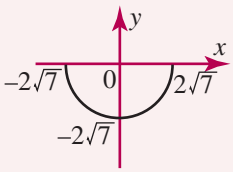
f 

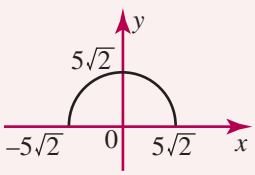
12 **a** 

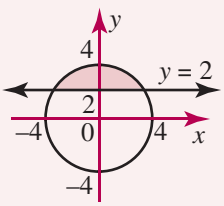
b 

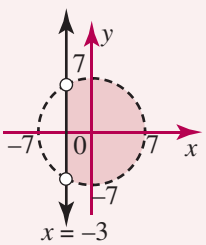
c 

d 

e 

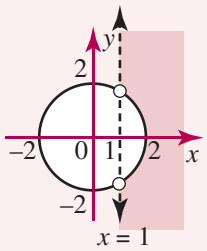
f 

13 **a** 

b 

ANSWERS

c



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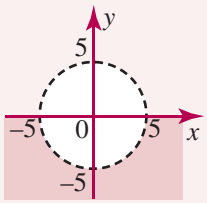

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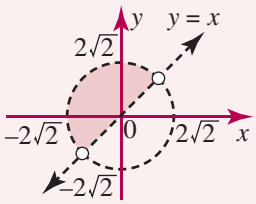

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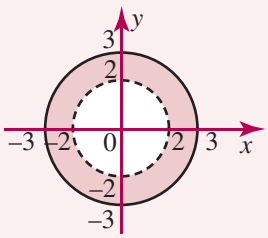
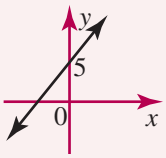


Exercise 7.13

1 **a** *L*, **b** *H*, **c** *P*, **d** *E*, **e** *C*, **f** *CU*, **g** *P*, **h** *L*, **i** *E*, **j** *H*, **k** *L*, **l** *C*, **m** *CU*, **n** *L*, **o** *H*, **p** *E*, **q** *C*, **r** *CU*, **s** *L*, **t** *P*, **u** *L*, **v** *H*, **w** *E*, **x** *C* **2** **a** $y = -3x - 1$, **b** $y = 3x - 1$, **c** $y = 1 - 3x$, **d** $y = 3x + 1$ **3** **a** $y = x^2 + 5$, **b** $y = x^2 - 5$, **c** $y = 5 - x^2$, **d** $y = -x^2 - 5$ **4** **a** $y = -\frac{2}{x}$, **b** $y = -\frac{x}{2}$, **c** $y = \frac{2}{x}$, **d** $y = \frac{x}{2}$ **5** **a** $y = 1 - x^3$, **b** $y = x^3 + 1$, **c** $y = x^3 - 1$, **d** $y = -x^3 - 1$ **6** **a** $y = -2^x$, **b** $y = 2^x$, **c** $y = 2^{-x}$, **d** $y = -2^{-x}$ **7** **a** *W*, **b** *Q*, **c** *N*, **d** *L*, **e** *R*, **f** *I*, **g** *S*, **h** *G*, **i** *B*, **j** *E*, **k** *V*, **l** *F*, **m** *M*, **n** *C*, **o** *O*, **p** *K*, **q** *A*, **r** *T*, **s** *P*, **t** *D*, **u** *U*, **v** *X*, **w** *H*, **x** *J*

8 **a**



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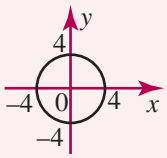

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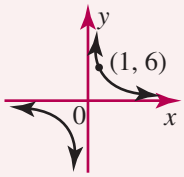

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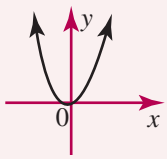

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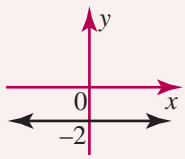

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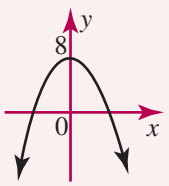

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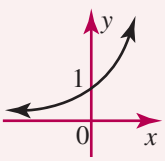

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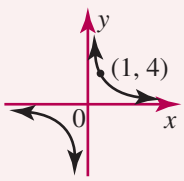
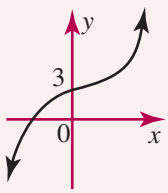


i



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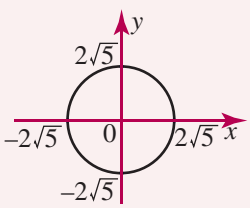

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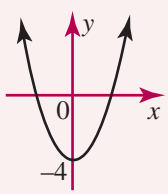
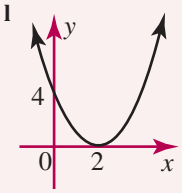
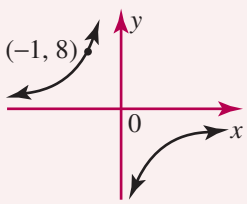


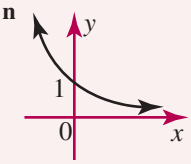
l



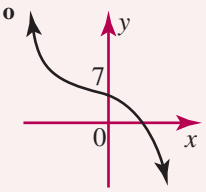
m



n



o



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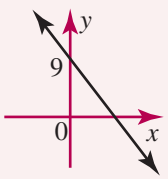

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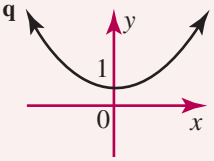

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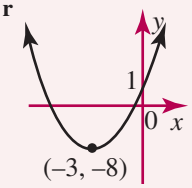

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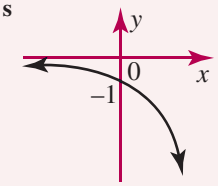

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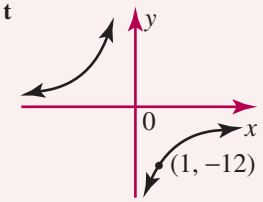
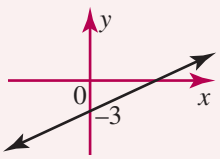


u



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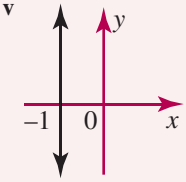
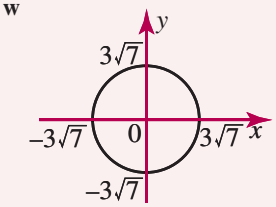
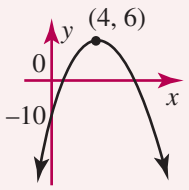


w



x



9 **a** 4 units, **b** (3, 4), **c** $k = 12$, $y = \frac{12}{x}$ **10** **a** $P(4, 5)$, $Q(10, 2)$, **b** $m = -\frac{1}{2}$ **11** **a** (6, 64), **b** (1, 1), (−1, 1), **c** (−2, 5), **d** (2, −4), (−2, −4), **e** (3, 3), (−3, −3), **f** (2, 4), **g** (5, 3), (5, −3), **h** (0, 0), (2, 2), (−2, −2), **i** (−1, −2), (3, 6)

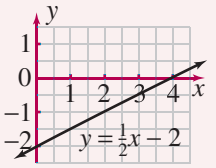
12 **a** $y = 6x - 2$, **b** $y = \frac{15}{x}$, **c** $y = 3x^2 + 3$, **d** $x^2 + y^2 = 36$, **e** $y = 2x^3 + 11$, **f** $y = 3(x + 1)(x - 5)$, **g** $y = -\frac{24}{x}$, **h** $y = -5x + 4$, **i** $y = 2^x$, **j** $y = 6 - x^2$, **k** $y = 5 - x^3$, **l** $y = 4(x - 1)^2 - 9$

Chapter 7 Review

1 **a** C, **b** B, **c** A, **d** C **2** **a** B, **b** C **3** B **4** C **5** A **6** **a** 9 am, **b** 140 km, **c** 10.30 am, **d** 12.30 pm, **e** 150 km, **f** 160 km, **g** Lloyd's speed decreased, the line is less steep, **h** 90 km, **i** 60 km/h, **j** 9.30 am

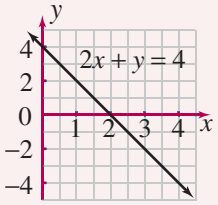
7 **a**

x	0	1	2	3	4
y	−2	$-1\frac{1}{2}$	−1	$-\frac{1}{2}$	0

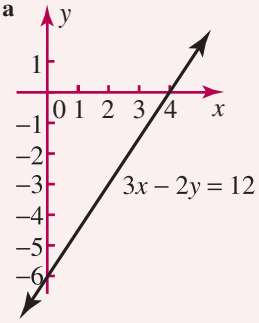


b

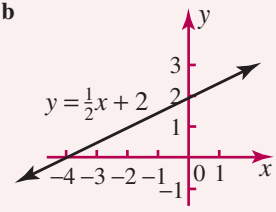
x	0	1	2	3	4
y	4	2	0	−2	−4



8 **a**

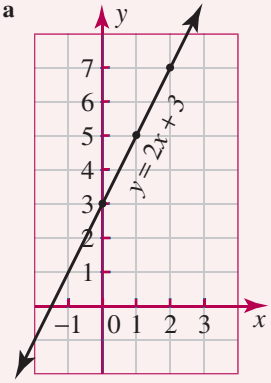


b

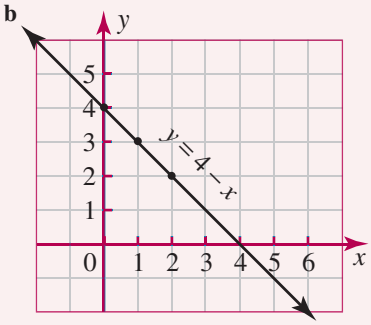


9 **a** $y = 4x - 3$, **b** $y = -2x$ **10** $-\frac{3}{4}$

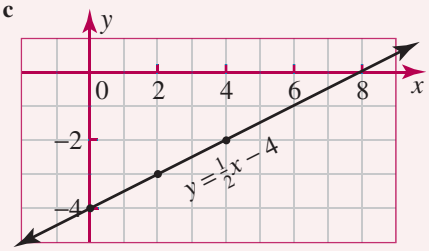
11 **a**



b



c

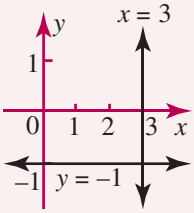


12 **a** $y = 7 - 2x$, **b** $y = 2x - 7$, **c** $y = -2x - 7$, **d** $y = 2x + 7$

13 **a** $y = -\frac{1}{5}x$, **b** $y = -5x$, **c** $y = 5x$, **d** $y = \frac{1}{5}x$

14 **a** $y = 4x + 20$, **b** $y = -\frac{3}{5}x - 6$

15



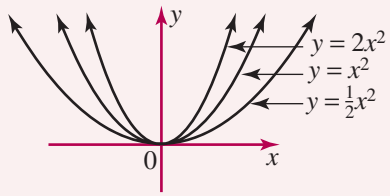
16 **a** $y = 5$, **b** $x = 4$

17 **a** yes, **b** no

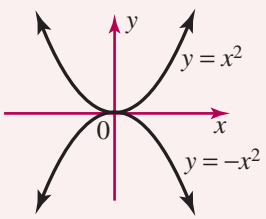
18 $k = 2$

19 **a** $y = x - 6$, **b** $y = 2x + 4$, **c** $y = 5 - x$, **d** $y = \frac{1}{3}x + 1$

20

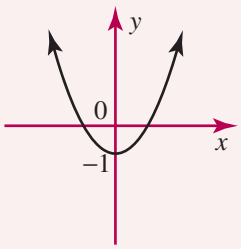


21

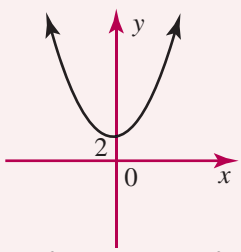


ANSWERS

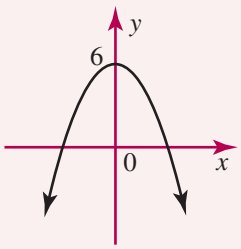
22 **a**



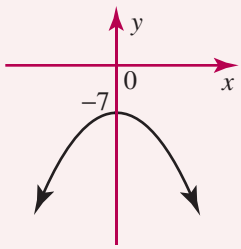
b



c

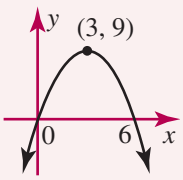


d

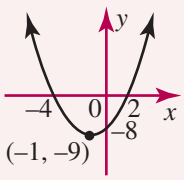


23 **a** $y = -x^2 - 1$, **b** $y = x^2 + 1$, **c** $y = x^2 - 1$, **d** $y = 1 - x^2$

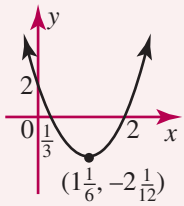
24 **a**



b

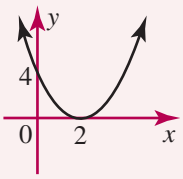


c

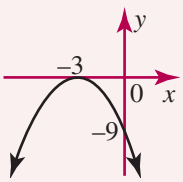


25 **a** $x = 4, (4, -6)$, **b** $x = 2, (2, 5)$ **26** −21

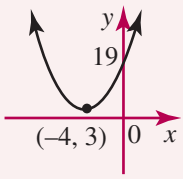
27 **a**



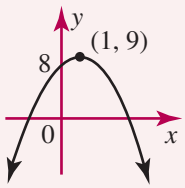
b



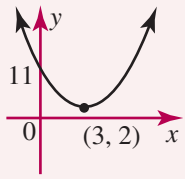
c



d

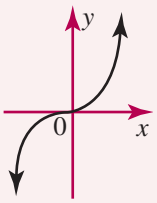


28

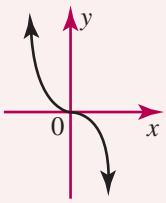


29 **a** $y = 4x^2$, **b** $y = x^2 - 5$ **30** $y = \frac{1}{2}x^2 - 7$

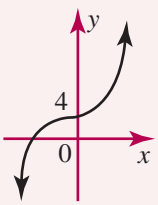
31 **a**



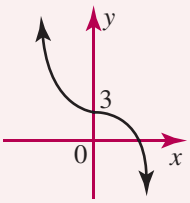
b



c

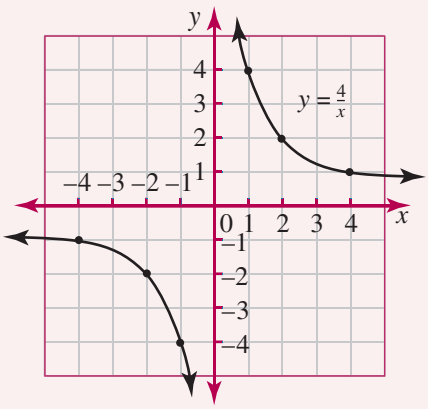


d

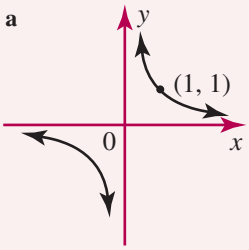


32

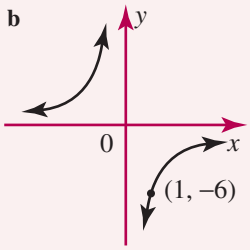
x	−4	−2	−1	0	1	2	4
y	−1	−2	−4	–	4	2	1



33 **a**

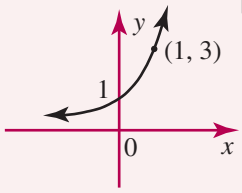


b

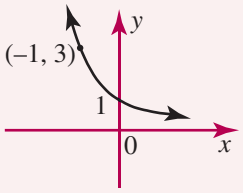


34 $t = -4, u = -\frac{2}{3}$ **35** **a** $y = \frac{24}{x}$, **b** $y = -\frac{9}{x}$ **36** **a** $y = \frac{x}{3}$, **b** $y = \frac{3}{x}$, **c** $y = -\frac{3}{x}$, **d** $y = -\frac{x}{3}$

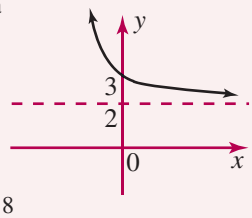
37 a



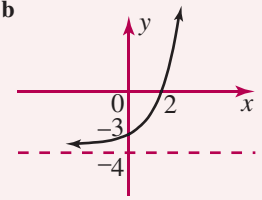
b



38 a



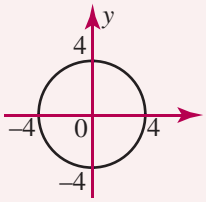
b



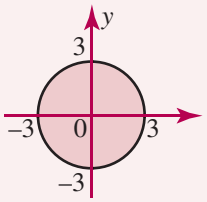
39 a $y = 3^x + 4$, **b** $y = 4^{-x}$

40 a $x^2 + y^2 = 25$, **b** $x^2 + y^2 = 18$

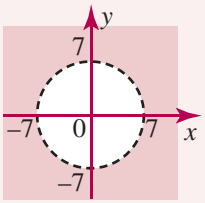
41



42 a



b



43 a $P(3, 11)$, $Q(-3, 11)$, **b** $P(2, 6)$, $Q(-2, -6)$

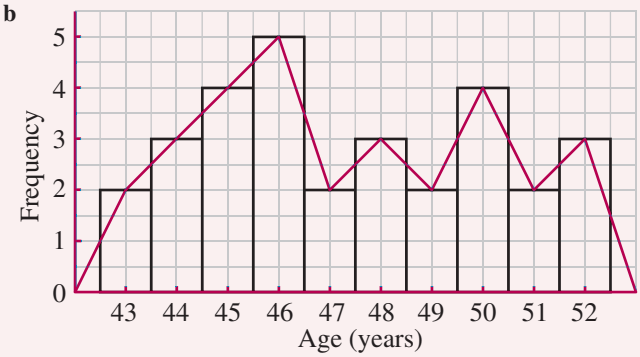
8 Data analysis and evaluation

Exercise 8.1

1 a

x	Tally	f	$c.f.$	fx
43	II	2	2	86
44	III	3	5	132
45	IIII	4	9	180
46	𝍸	5	14	230
47	II	2	16	94
48	III	3	19	144
49	II	2	21	98
50	IIII	4	25	200
51	II	2	27	102
52	III	3	30	156
		30		1422

b



c 19, **d** 13.3%

2 a 7, **b** 25, **c** 7, **d** 40, **e** 12.5% **3 a** 5, **b** 8, **c** 1, **d** Yes, 8, **e** Yes, 2–4, **f** 36% **4 a** 30, **b** 202 cm, 164 cm, **c** 184 cm, **d** 18
5 a 15.6, **b** 29.6, **c** 4.8, **d** 11.8 **6 a** 11, **b** 28, **c** 39, **d** 78.9
7 a 4, **b** 10, **c** 31, 37, **d** no mode **8 a** 27, **b** 33, **c** 34.2, **d** 51

9 a mean = 11, median = 10, mode = 16, range = 25, **b** mean = 26.5, median = 28.5, mode = 29, range = 25, **c** mean = 59.3, median = 59, mode = 62, range = 19, **d** mean = 35.7, median = 33.5, mode = 32, range = 15, **e** mean = 124.7, median = 125, mode = 117, 134, range = 20, **f** mean = 95.1, median = 95, mode = 94, 97, range = 8, **g** mean = 8, median = 8.1, mode = 8.3, range = 1.5, **h** mean = 13.9, median = 14.0, no mode, range = 1.6

10 a mean = 4.4, median = 5, mode = 6, range = 5, **b** mean = 13.7, median = 13, mode = 12, range = 5, **c** mean = 19.0, median = 19, mode = 15, range = 24, **d** mean = 39.6, median = 40, mode = 38, range = 22

11 a

x	f	$c.f.$	fx
17	4	4	68
18	2	6	36
19	4	10	76
20	3	13	60
21	6	19	126
22	5	24	110
23	3	27	69
	27		545

b mean = 20.2, median = 21, mode = 21, range = 6

12 a 43, **b** 7, **c** 16 yrs, **d** 5 yrs, **e** 15 yrs, **f** 14.9 yrs

13 a

Stem	Leaf
$0^{(5)}$	8 9
$1^{(0)}$	0 2 4
$1^{(5)}$	5 6 6 7 8 9 9
$2^{(0)}$	0 2 3 4 4
$2^{(5)}$	5 5 5 6 7 7 9 9
$3^{(0)}$	0 1 2 4
$3^{(5)}$	6

b 30, **c** $36 000, **d** $28 000, **e** $25 000, **f** $23 500, **g** $22 100, **h** 53%

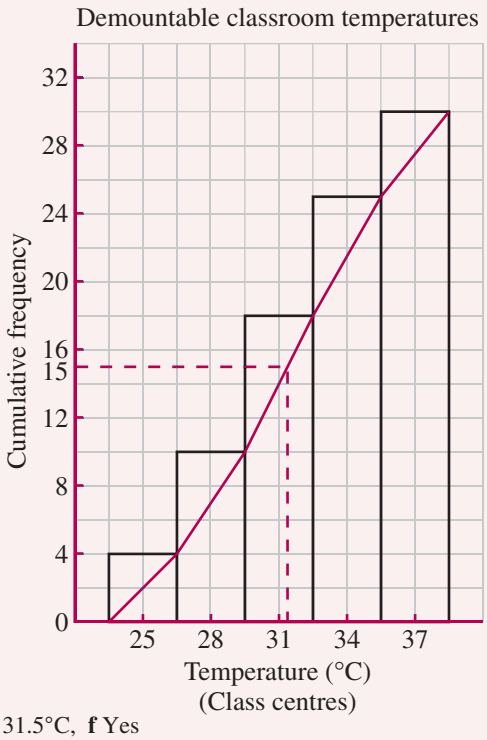
14 a 10, **b** 26, **c** 2.6, **d** 4, **e** 7th, **f** 4th
15 a 14, **b** 7.5

16 a

Class	Class centre (*c.c.*)	Tally	Frequency (*f*)	$f \times c.c.$
24–26	25	IIII	4	100
27–29	28	𝍸 I	6	168
30–32	31	𝍸 III	8	248
33–35	34	𝍸 II	7	238
36–38	37	𝍸	5	185
			30	939

b 30–32, **c** 31.3°C

d



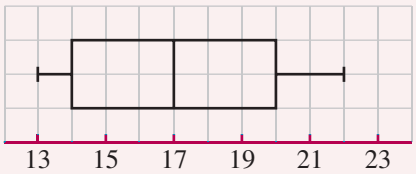
e 31.5°C, **f** Yes

17 a 17–21, 22–26, 27–31, 32–36, 37–41, 42–46, **b** 30, **c** 27–31, **d** 21, **e** 32 yrs **18** 4
19 a 17, **b** 216, **c** 28 **20 a** 25, **b** 8
21 a decrease, **b** increase, **c** stay the same
22 a stay the same, **b** increase, **c** decrease
23 a 61.2, **b** 62.5 **24** 63 **25** 92%
26 79% **27** 69.1 **28 a** 53, **b** 29
29 a 16, **b** 95 **30** $x = 5$, $y = 8$

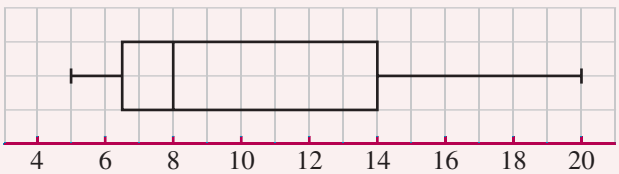
Exercise 8.2

1 a 66, **b** 37 **2 a** 42, **b** 65 **3 a i** 2, 12, **ii** 10, **iii** 7, **iv** 4, 9, **v** 5, **b i** 0, 20, **ii** 20, **iii** 7, **iv** 3, 15, **v** 12,
4 a i 8, **ii** 13, **iii** 10, 14, **iv** 4, **b i** 20, **ii** 27, **iii** 19, 31.5, **iv** 12.5, **c i** 27, **ii** 47, **iii** 38, 52, **iv** 14, **d i** 29,
ii 39.5, **iii** 30.5, 49, **iv** 18.5, **e i** 32, **ii** 66, **iii** 55, 70, **iv** 15, **f i** 36, **ii** 45.5, **iii** 38, 60, **iv** 22

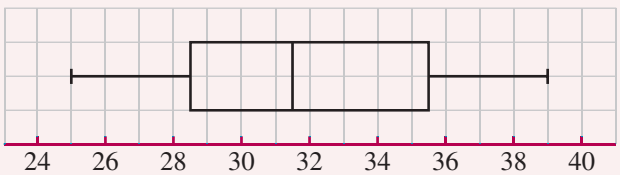
5 a median = 17, quartiles = 14, 20



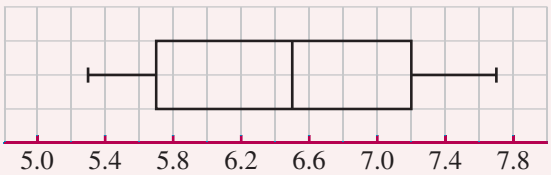
b median = 8, quartiles = 6.5, 14



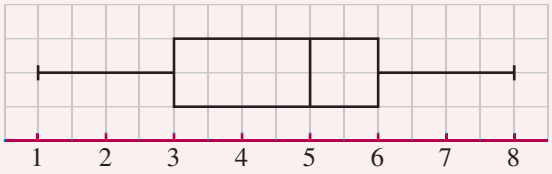
c median = 31.5, quartiles = 28.5, 35.5



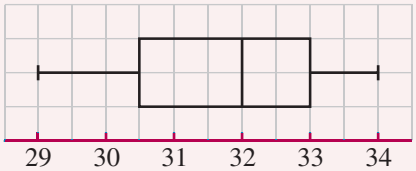
d median = 6.5, quartiles = 5.7, 7.2



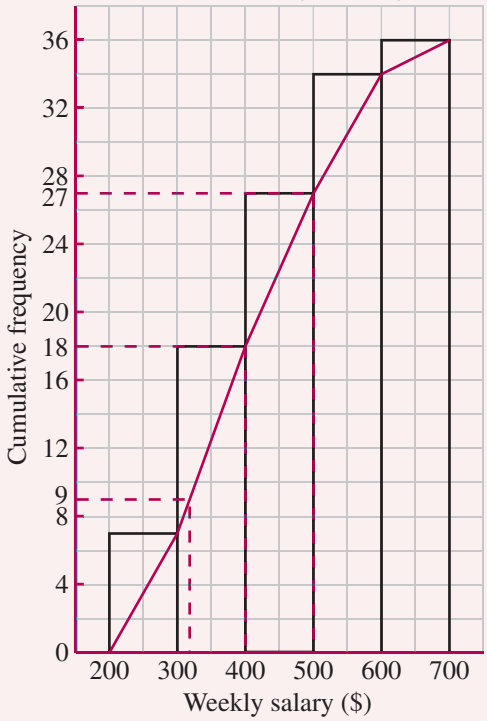
6 **a** median = 5, IQR = 3, **b**



7 **a** **i** 25%, **ii** 25%, **iii** 50%, **iv** 75%, **b** 7 **8** 84 **9** 18 **10** **a** extremes = 3 yrs, 9 yrs, range = 6 yrs, **b** 5.5 yrs, **c** 4 yrs, 7 yrs, **d** 3 yrs **11** **a** 5, **b** 6.5 **12** No, the median is the middle score of all the scores not the middle score between the quartiles. **13** **a** 32, **b** 30.5, 33, **c** 2.5,
d



14 **a** median = 49.5, IQR = 2, **b** median = 4, IQR = 2.5 **15** **a** 46 yrs, **b** 76 yrs, **c** **i** 18–33 yrs, **ii** 33–55 yrs
16 **a** F, **b** T, **c** F, **d** T, **e** F **17** **a**



b $400, **c** $180 **19** 1

Exercise 8.3

1 **a** **i** 20, **ii** $\sum(x-\bar{x})^2 = 124$, **iii** 5.0, **b** **i** 59, **ii** $\sum(x-\bar{x})^2 = 134$, **iii** 4.7 **2** **a** $\bar{x} = 5.9, \sigma_n = 1.9$, **b** $\bar{x} = 17, \sigma_n = 8.0$, **c** $\bar{x} = 20.2, \sigma_n = 6.2$, **d** $\bar{x} = 34.3, \sigma_n = 5.0$, **e** $\bar{x} = 17.0, \sigma_n = 7.9$, **f** $\bar{x} = 4.7, \sigma_n = 2.7$ **3** **a** **i** 11.7, **ii** 12, **iii** 13, **iv** 1.5, **b** **i** 97.7, **ii** 97, **iii** 96, **iv** 1.6, **c** **i** 31.9, **ii** 32.5, **iii** 35, **iv** 6.6 **4** **a** range = 22, $\sigma_n = 6.7$, **b** range = 4, $\sigma_n = 1.5$, **c** range = 7, $\sigma_n = 1.9$, **d** range = 4, $\sigma_n = 1.2$
5 **a** 5 yrs, **b** 24 yrs, **c** 23.5 yrs, **d** 22.7 yrs, **e** 1.8 yrs **6** **a** $\bar{x} = 28.4, \sigma_n = 13.9$, **b** $\bar{x} = 25, \sigma_n = 10.6$
7 **a** 11°C, **b** 4.5°C, **c** 2.9°C, **d** The range and standard deviation, since there are no outliers. **8** **a** 136, **b** 17, **c** 27.4, **d** The interquartile range, since the range and standard deviation are affected by outliers (9 and 145).
9 In Lucy's class — the standard deviation is smaller in her class. **10** **a** F, **b** F, **c** T **11** $\sigma_n = 0$, the scores are all equal to the mean (i.e. $\sum(x-\bar{x})^2 = 0$, $\therefore \sigma_n = 0$). **12** **a** 4.4, **b** The standard deviation would increase because the extra score is a long way from the mean. **13** **a** Wombats: $\bar{x} = 95.2, \sigma_n = 7.2$,
Ferrets: $\bar{x} = 95.5, \sigma_n = 4.6$, **b** The Ferrets, the standard deviation of the Ferrets scores is less than the standard deviation of the Wombats scores. **14** **a** Mrs Bremner's class: $\bar{x} = 78.3, \sigma_n = 11.1$, Mrs Spencer's class: $\bar{x} = 77.8, \sigma_n = 8.8$, **b** Mrs Bremner's class — the mean is higher for her class.

c Mrs Spencer's class — the standard deviation is lower for her class. **15** **a** $\bar{x} = 4, \sigma_n = 2$, **b** The mean would increase by 5, the standard deviation would not change. **c** The mean and standard deviation would both be 5 times greater. **16** The standard deviation would not change since the spread of both sets of scores is the same. That is, the average distance from the mean is the same in each set.

Exercise 8.4

1 **a** 68, **b** 52, **c** 76, **d** 44, **e** 84, **f** 36 **2** **a** English: 46, Drama: 45, **b** English: 88, Drama: 69, **c** English: 67, Drama: 57, **d** English: 39, Drama: 41 **3** **a** 45, **b** 69, **c** 33, **d** 81, **e** 63, **f** 54, **g** 65, **h** 47, **i** 39, **j** 71, **k** 24, **l** 89 **4** 7 **5** **a** 6, **b** 65, **c** 74, **d** 50 **6** **a** 1 s.d. above $\bar{x}$, **b** 2 s.d. below $\bar{x}$, **c** $1\frac{1}{2}$ s.d. below $\bar{x}$

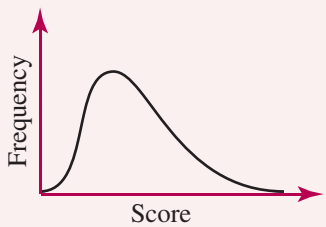
7 Science, the Science mark is $1\frac{1}{2}$ s.d. above the mean but the Maths mark is only 1 s.d. above the mean.

8 **a** Test 2, **b** Test 1, **c** Test 1, **d** Test 2, **e** Test 1, **f** Test 2, **g** Test 1, **h** Test 1 **9** **a** **i** 80, **ii** 56, **iii** 92, **iv** 44, **b** **i** 74, **ii** 66, **iii** 82, **iv** 60 **10** No, she will fail. **11** **a** 58 g, 72 g, **b** 51 g, 79 g, **c** 44 g, 86 g
12 **a** 68%, **b** 95%, **c** 34%, **d** 13.5%, **e** 81.5%, **f** 16% **13** **a** 272, **b** 380, **c** 136, **d** 326, **e** 64, **f** 10
14 \$340–\$800 **15** 10.2–13.8 km/L **16** **a** 2.5%, **b** No, the mark would be 4 s.d. above the mean.
17 **a** $\bar{x}$ will increase, σ_n will increase, **b** $\bar{x}$ will decrease, σ_n will increase, **c** $\bar{x}$ will stay the same, σ_n will decrease
18 **a** $\bar{x}$ will decrease, σ_n will decrease, **b** $\bar{x}$ will increase, σ_n will decrease, **c** $\bar{x}$ will stay the same, σ_n will increase

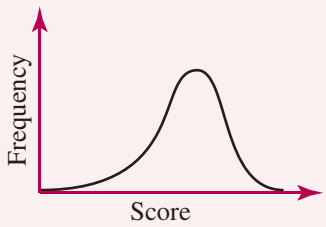
Exercise 8.5

1 **a**

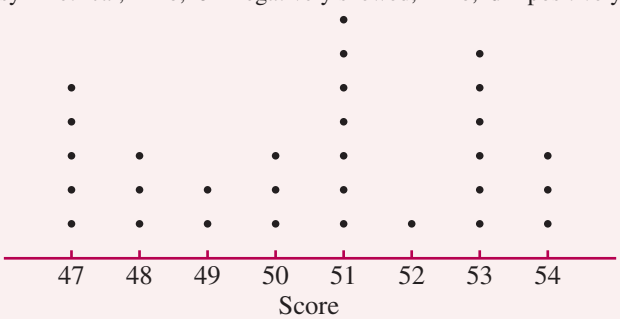
b



c

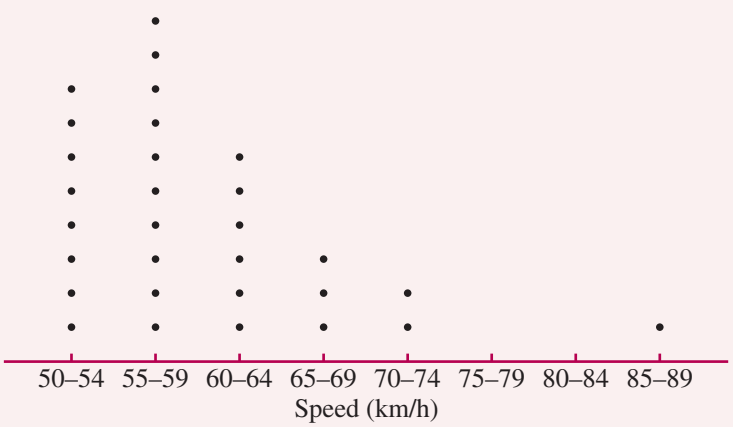


2 **a** negatively skewed, **b** positively skewed, **c** symmetrical **3** **a** **i** positively skewed, **ii** yes, 9
b **i** symmetrical, **ii** no, **c** **i** negatively skewed, **ii** no, **d** **i** positively skewed, **ii** yes, 8 **4** Positively skewed
5 **a**



b 3, **c** 51
6 **a** negatively skewed, **b** symmetrical

7 **a**



b Yes, 88,
c positively skewed,
d 6

8 No, skewness only applies to numerical or quantitative data, not categorical data.

9 a

Stem	Leaf
0	3 4 6 8
1	2 3 3 5 6 7
2	0 1 2 4 4 7 7 8
3	4 5 6 9
4	0 1 5
5	2 4
6	1

b No, **c** Positively skewed, **d** No, most of his scores are low.

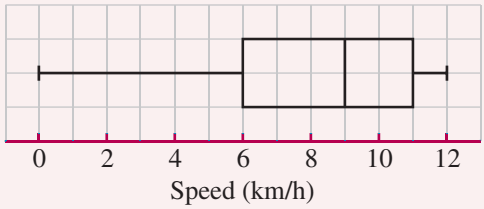
10 a Negatively skewed, **b** Positively skewed, **c** Symmetrical

11 a

Absentees	0	1	2	3	4	5	6	7	8	9	10	11	12
Frequency	1	1	2	2	2	3	3	3	3	5	9	8	6

b lowest score = 0, highest score = 12, median = 9, lower quartile = 6, upper quartile = 11.

c



d negatively skewed

12 a

Stem	Leaf
$7^{(5)}$	7 8
$8^{(0)}$	1 2 2 3 3 4 4 4 4
$8^{(5)}$	5 5 6 6 7
$9^{(0)}$	0 0 1 1 2 2 2 3 4
$9^{(5)}$	5 6 6 7 9

b Bimodal, **c** No

13 a i positively skewed, **ii** negatively skewed, **b** no skew

14 a 0.47, **b** –0.43

Exercise 8.6

1 a i 36, **ii** 43, **b** 12, **c** Monday, **d** 73, **e** Monday, **f** 10% **2 a i** August, **ii** January, **iii** September, **iv** December, **b** September, October, July, **c** January, February, March, December, **d** January, February, December **3 a i** 4000, **ii** 4500, **iii** 1750, **b i** 3000, **ii** 4500, **iii** 3750, **c i** Females, **ii** Males, **d i** 8250, **ii** 17 500, **e** 1500 **4 a** 10 m, **b** 15 m, **c i** 25 m, **ii** 54 m, **iii** 80 m, **d** 4 m, **e** 46 m **5 a** Alice Springs: mean = 28.4°C, median = 29.3°C, range = 16.7°C, IQR = 11.5°C; Darwin: mean = 31.6°C, median = 31.7°C, range = 3.5°C, IQR = 1.6°C. **b** Yes, although Jan, Feb, Dec are hotter months in Alice Springs, the temperatures are more consistently high in Darwin throughout the year. **6 a** Yr 9, **b** Yr 8, **c** Yr 7, **d** Yr 10, **e** Yr 9, **f** Yr 8 **7 a** Sam, **b** Bob, **c** Ros, **d** Bob **8 a** Yr 7: lowest score = 20, highest score is 90, median = 55, lower quartile = 30, upper quartile = 70; Yr 8: lowest score = 40, highest score is 95, median = 70, lower quartile = 55, upper quartile = 80. **b** Yr 7 — 40, Yr 8 — 25, **c** The students results have improved greatly. In Yr 7 50% of the marks were greater than 55, in Yr 8 75% of the marks were greater than 55.

9 a

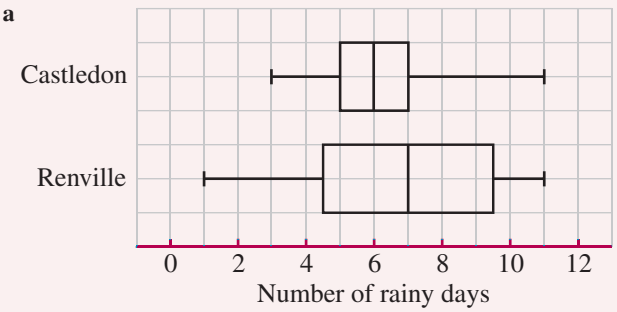
First round	Stem	Second round
9 8 7	$6^{(5)}$	5 7 7 7
4 4 3 2 2 2 2 0 0	$7^{(0)}$	0 0 1 1 1 2 3 3 4 4
9 8 8 7 6 5 5 5	$7^{(5)}$	5 5 6 6 7 8 9 9
4 3 1 0	$8^{(0)}$	2
	$8^{(5)}$	5

b Round 1: mean = 74.75, median = 74.5, mode = 72; Round 2: mean = 73.63, median = 74, mode = 67, 71

c Mean, since there are no outliers.

d Second round

10 a



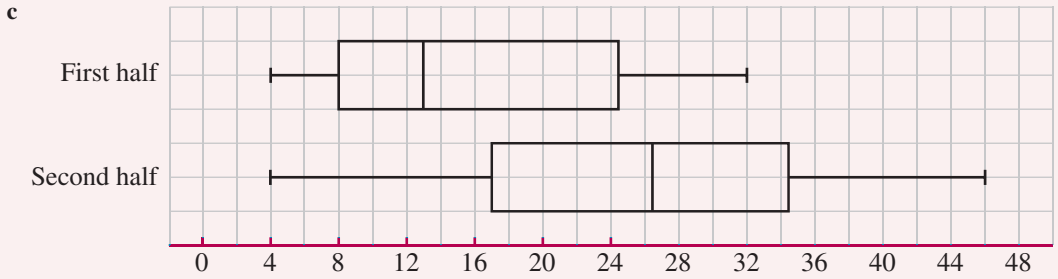
b Castledon — 2.1, Renville — 3.0,

c Renville has 6 more rainy days (80) compared to Castledon (74), and the rainfall pattern is more wide spread.

11 a 20, **b i** $\bar{x} = 15.85$, $\sigma_n = 8.57$,

ii $\bar{x} = 25.55$, $\sigma_n = 11.20$

c



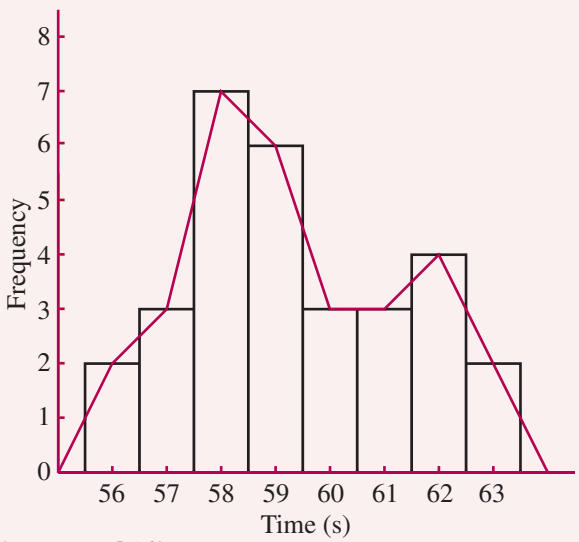
d Overall, the team concedes more points in the second half. **12 a** The means are all 7, **b i** C, **ii** A
c A: $\bar{x} = 7, \sigma_n = 1.1$; B: $\bar{x} = 7, \sigma_n = 1.4$; C: $\bar{x} = 7, \sigma_n = 1.6$

Chapter 8 Review

1 a

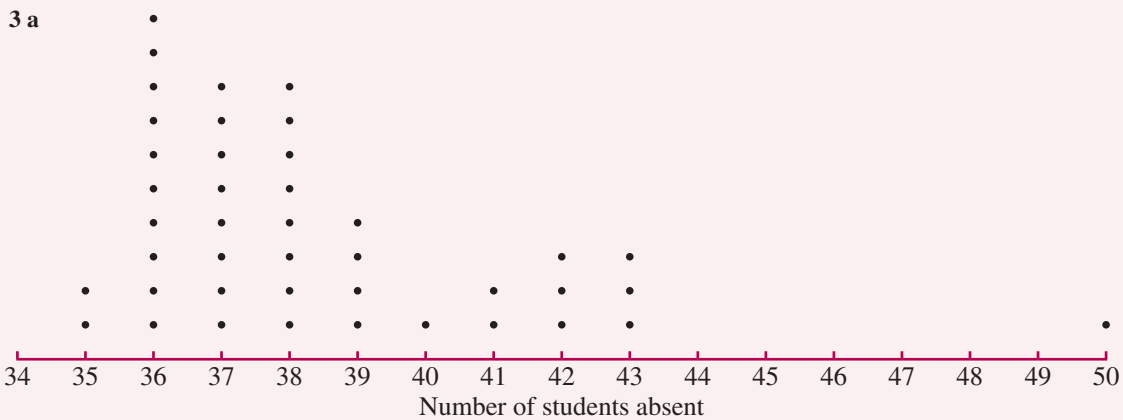
x	Tally	*f*	*c.f.*	*fx*
56	II	2	2	112
57	III	3	5	171
58	𝍸 II	7	12	406
59	𝍸 I	6	18	354
60	III	3	21	180
61	III	3	24	183
62	IIII	4	28	248
63	II	2	30	126
		30		1780

b



c 18, **d** 20%, **e** 63 s, **f** 58 s **2 a** 27, **b** 189 cm, 148 cm, **c** 7, **d** 169.7 cm

3 a



b yes, 36–38, **c** yes, 50
4 a mean = 14, median = 15, mode = 18, range = 17, $\sigma_n = 5.5$, **b** mean = 46, median = 45, mode = 59, range = 68, $\sigma_n = 21.3$, **c** mean = 34.5, median = 35, mode = 30, 37, range = 9, $\sigma_n = 3.1$, **d** mean = 5.4, median = 5.1, no mode, range = 3.8, $\sigma_n = 1.3$ **5 a** mean = 3.8, median = 4, mode = 5, range = 4, $\sigma_n = 1.3$, **b** mean = 26.7, median = 27, mode = 23, range = 15, $\sigma_n = 5.1$ **6 a** 6, **b** 17 yrs, **c** 29, **d** 6.9%, **e** 15 yrs, **f** 14 yrs, **g** 14.2 yrs **7 a** 82, **b** 54

8 **a** 12, **b** 1, **c** 4 yrs, **d** 72.5 yrs, **e** 73 yrs, **f** 72.1 yrs, **g** 1.4 yrs

9 **a**

Class	Class centre (*c.c.*)	Tally	Frequency (f)	$f \times c.c.$
1–5	3	I	1	3
6–10	8	I	1	8
11–15	13	IIII	4	52
16–20	18	𝍸 III	8	144
21–25	23	𝍸 𝍸 II	12	276
26–30	28	IIII	4	112
			30	595

b 21–25, **c** 14, **d** $\bar{x} = 19.8$, $\sigma_n = 5.8$

10 **a** 112, **b** 32 **11** 10

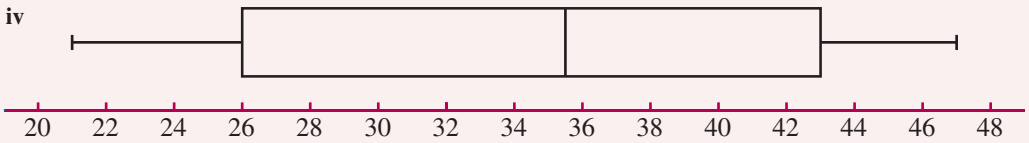
12 **a** **i** increase, **ii** decrease,
b **i** decrease, **ii** increase,
c **i** stay the same, **ii** stay the same

13 **a** 70.1, **b** 71.4 **14** 92% **15** 50

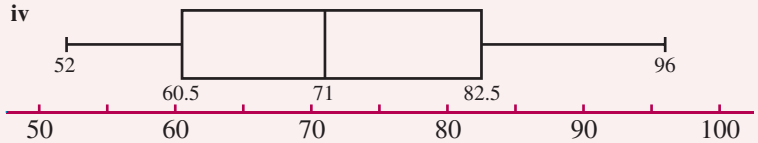
16 **a** **i** L.S. = 1, H.S. = 9, **ii** 6, **iii** 8, **iv** 3, **v** 8, **vi** 5, **b** **i** 25%, **ii** 25%, **iii** 50%, **iv** 25%

17 **a** **i** 10, **ii** $Q_1 = 6$, $Q_3 = 15$, **iii** 9, **iv**

b **i** 35.5, **ii** $Q_1 = 26$, $Q_3 = 43$, **iii** 17, **iv**



c **i** 71, **ii** $Q_1 = 60.5$, $Q_3 = 82.5$, **iii** 22, **iv**



18 $Q_1 = 3$, $Q_3 = 6$, I.Q.R. = 3 **19** **a** 46, **b** $Q_1 = 44$, $Q_3 = 46.5$, **c** 2.5 **20** A **21** B

22 **a** Petersham: $\bar{x} = 224.4$, $\sigma_n = 43.0$; Manly: $\bar{x} = 242.8$, $\sigma_n = 82.9$, **b** The Petersham team had a lower standard deviation, so they were the more consistent team. **23** **a** 62, **b** 26, **c** 56, **d** 32 **24** 6

25 **a** test 1, **b** test 2 **26** **a** **i** 78, **ii** 54, **b** **i** 58, **ii** 88 **27** **a** 68%, **b** 95%, **c** 99.7%, **d** 34%, **e** 13.5%, **f** 2.5%

28 **a** 136, **b** 190, **c** 5% **29** **a** positively skewed, **b** negatively skewed, **c** symmetrical

30 **a** Boys: $\bar{x} = 75.1$, median = 74.5, $\sigma_n = 7.4$; Girls: $\bar{x} = 72.1$, median = 73.5, $\sigma_n = 7.5$

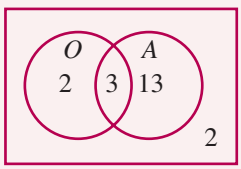
b The boys have a higher mean and median so they are slightly fitter than the girls.

9 Probability

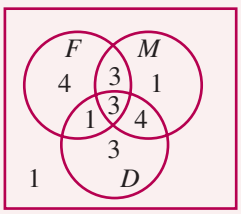
Exercise 9.1

1 **a** $\frac{1}{13}$, **b** $\frac{1}{2}$, **c** $\frac{1}{26}$, **d** $\frac{4}{13}$, **e** $\frac{3}{13}$ **2** **a** $\frac{3}{10}$, **b** $\frac{3}{10}$, **c** $\frac{2}{5}$, **d** $\frac{2}{5}$ **3** **a** **i** $\frac{18}{25}$, **ii** $\frac{1}{10}$, **iii** $\frac{9}{50}$, **b** yes

4 **a** **i** A and B, D and F, **ii** A and B, C and E, D and F, E and F, **b** $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2}$, $P(C) = \frac{2}{5}$, $P(D) = \frac{3}{5}$, $P(E) = \frac{3}{10}$, $P(F) = \frac{2}{5}$ **5** **a** $\frac{2}{5}$, **b** $\frac{1}{5}$, **c** $\frac{4}{5}$, **d** 0 **6** **a** $\frac{1}{30}$, **b** $\frac{1}{10}$, **c** $\frac{1}{6}$, **d** $\frac{19}{30}$, **e** $\frac{7}{30}$ **7** **a** **i** $\frac{3}{14}$, **ii** $\frac{13}{28}$, **iii** $\frac{11}{28}$, **iv** $\frac{1}{7}$, **b** **i** $\frac{7}{11}$, **ii** $\frac{11}{15}$ **8** **a** $\frac{7}{8}$, **b** $\frac{13}{40}$, **c** $\frac{27}{40}$, **d** $\frac{23}{40}$, **e** $\frac{1}{10}$ **9** **a** $\frac{1}{12}$, **b** $\frac{1}{3}$, **c** $\frac{1}{12}$, **d** $\frac{1}{12}$, **e** $\frac{1}{6}$, **f** $\frac{1}{4}$, **g** $\frac{1}{3}$ **10** $P(T) = \frac{18}{25}$, $P(S) = \frac{2}{5}$, $P(S \text{ and } T) = \frac{3}{25}$ **11**



12 **a** **i** $\frac{2}{9}$, **ii** $\frac{1}{9}$, **b** $\frac{2}{3}$, **c** $\frac{1}{9}$, **d** 6 or 8 **15**



13 **a** $P(T) = \frac{13}{20}$, $P(H) = \frac{7}{20}$, the coin is biased towards tails.

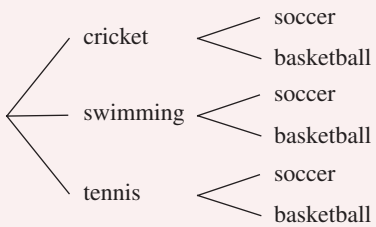
c toss the coin a large number of times.

ANSWERS

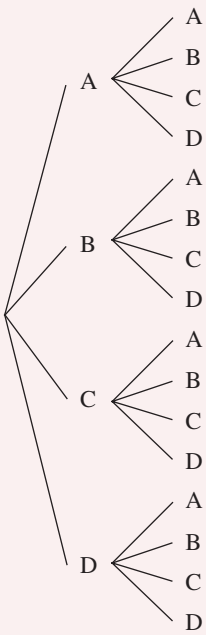
Exercise 9.2

1 a i $\frac{2}{9}$, **ii** $\frac{1}{3}$, **iii** $\frac{1}{9}$, **iv** $\frac{2}{3}$, **b i** $\frac{1}{18}$, **ii** $\frac{1}{6}$, **iii** $\frac{1}{6}$, **iv** $\frac{2}{3}$, **v** $\frac{1}{3}$

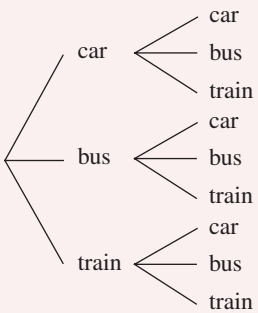
2 a



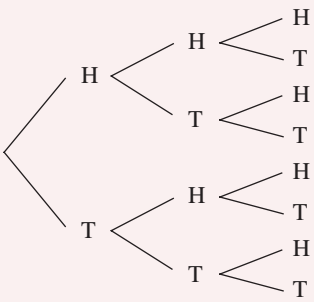
b



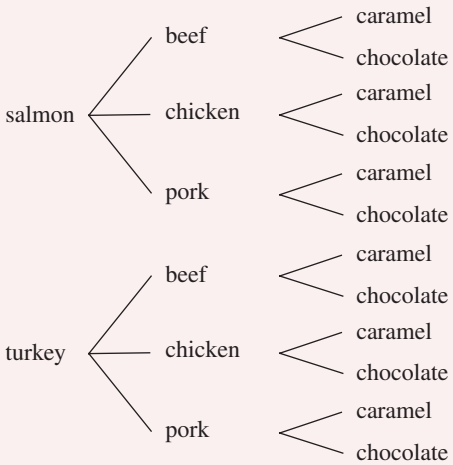
c



d



e



3 a i $\frac{1}{9}$, **ii** $\frac{2}{9}$, **iii** $\frac{4}{9}$, **iv** $\frac{5}{9}$, **b i** $\frac{1}{4}$, **ii** $\frac{1}{4}$, **iii** $\frac{1}{2}$, **c i** $P(B) = \frac{1}{4}$, $P(J) = \frac{1}{4}$, $P(D) = \frac{1}{2}$, **ii** $\frac{1}{8}$, **iii** $\frac{3}{4}$, **d i** $\frac{1}{3}$, **ii** $\frac{1}{6}$, **iii** $\frac{1}{3}$

4 a i $\frac{1}{18}$, **ii** $\frac{1}{36}$, **iii** $\frac{1}{9}$, **iv** 0, **v** $\frac{1}{3}$, **vi** $\frac{13}{18}$, **b i** $\frac{1}{8}$, **ii** $\frac{1}{56}$, **iii** $\frac{3}{56}$, **iv** $\frac{3}{7}$, **v** $\frac{2}{7}$ **5 a** 24, **i** $\frac{1}{24}$, **ii** $\frac{1}{2}$, **iii** $\frac{1}{12}$, **iv** $\frac{7}{24}$, **v** $\frac{5}{8}$

b i $\frac{1}{36}$, **ii** $\frac{1}{36}$, **iii** $\frac{1}{18}$, **iv** $\frac{2}{3}$, **v** $\frac{5}{6}$, **c i** $\frac{1}{16}$, **ii** $\frac{1}{4}$, **iii** $\frac{1}{2}$, **iv** $\frac{1}{4}$, **v** $\frac{1}{16}$, **vi** $\frac{7}{16}$, **d i** $\frac{1}{24}$, **ii** $\frac{3}{8}$, **iii** $\frac{2}{3}$, **iv** $\frac{1}{12}$, **v** $\frac{1}{3}$, **vi** $\frac{2}{3}$

7 a i $\frac{1}{49}$, **ii** $\frac{1}{49}$, **iii** $\frac{6}{49}$, **iv** $\frac{36}{49}$, **v** $\frac{5}{7}$, **b i** 1, **ii** $\frac{2}{25}$, **iii** $\frac{3}{5}$, **iv** $\frac{2}{5}$, **v** $\frac{1}{5}$, **c i** $\frac{1}{144}$, **ii** $\frac{1}{12}$, **iii** $\frac{5}{24}$, **iv** $\frac{1}{9}$, **v** $\frac{35}{72}$, **vi** $\frac{47}{48}$,

d i $\frac{1}{2}$, **ii** $\frac{1}{52}$, **iii** $\frac{1}{26}$, **iv** $\frac{15}{26}$, **v** $\frac{1}{169}$ **8 a** $\frac{1}{64}$, **b** $\frac{27}{2197}$, **c** $\frac{1}{2704}$, **d** $\frac{1728}{2197}$, **e** $\frac{27}{4394}$ **9 a** $\frac{1}{81}$, **b** $\frac{1}{36}$, **c** $\frac{25}{1296}$, **d** $\frac{1}{72}$,

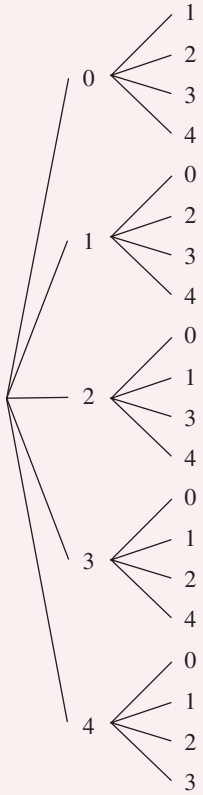
e $\frac{16}{81}$ **10 a** $\frac{1}{216}$, **b** $\frac{1}{1728}$, **c** $\frac{1}{72}$, **d** $\frac{8}{27}$, **e** $\frac{271}{288}$ **11 a** 1296, **b i** $\frac{1}{6}$, **ii** $\frac{1}{6}$, **iii** $\frac{1}{16}$, **iv** $\frac{1}{27}$, **v** $\frac{13}{18}$ **12 a** $\frac{351}{400}$,

b $\frac{23}{100}$, **c** $\frac{83}{400}$, **d** $\frac{39}{400}$ **13 a** 281 474 976 710 656, **b** 2.8%, **c** 1.7%, **d** 88.5%

Exercise 9.3

1 **a** AB, AC, AD, BC, BD, CD, **b** **i** $\frac{1}{6}$, **ii** $\frac{1}{2}$, **iii** $\frac{1}{2}$

2 **a**



, **b** **i** $\frac{1}{20}$, **ii** $\frac{4}{5}$, **iii** $\frac{2}{5}$, **iv** $\frac{1}{5}$, **v** $\frac{3}{10}$

3 **a** 90, **b** 81, **c** **i** $\frac{1}{5}$, **ii** $\frac{22}{45}$, **iii** $\frac{4}{45}$, **iv** $\frac{22}{45}$

4 **a** $\frac{2}{5}$, **b** $\frac{1}{10}$, **c** $\frac{3}{10}$, **d** $\frac{3}{10}$, **e** $\frac{9}{10}$

5 **a** 1128, **b** **i** $\frac{35}{376}$, **ii** $\frac{22}{47}$, **iii** $\frac{25}{47}$, **c** $\frac{24}{47}$

6 **a** **i** $\frac{1}{5}$, **ii** $\frac{26}{35}$, **iii** $\frac{8}{105}$, **b** **i** 0, **ii** $\frac{4}{455}$, **iii** $\frac{8}{65}$, **iv** $\frac{13}{35}$

7 **a** **i** $\frac{1}{100}$, **ii** $\frac{1}{100}$, **b** **i** $\frac{1}{20}$, **ii** $\frac{1}{20}$, **iii** $\frac{1}{20}$

8 **a** **i** $\frac{1}{66}$, **ii** $\frac{6}{11}$, **iii** $\frac{6}{11}$, **b** **i** $\frac{1}{220}$, **ii** $\frac{21}{55}$, **iii** $\frac{1}{440}$, **iv** $\frac{3}{220}$, **c** **i** $\frac{27}{64}$, **ii** $\frac{1}{576}$, **iii** $\frac{11}{144}$

9 $\frac{1}{3}$

10 **a** $\frac{1}{3}$, **b** $\frac{1}{2}$, **c** $\frac{2}{5}$, **d** $\frac{3}{5}$

11 **a** **i** $\frac{1}{51}$, **ii** $\frac{1}{51}$, **iii** $\frac{1}{1275}$, **iv** $\frac{1}{425}$, **b** **i** $\frac{1}{51}$, **ii** $\frac{2500}{132\,651}$, **iii** $\frac{2}{2601}$, **iv** $\frac{100}{44\,217}$

12 hard-hard or hard-soft

13 **a** $\frac{33}{16\,660}$, **b** $\frac{64}{162\,435}$, **c** $\frac{1}{54\,145}$, **d** $\frac{1}{108\,290}$

14 **a** $\frac{14}{495}$, **b** $\frac{49}{825}$, **c** $\frac{49}{825}$, **d** $\frac{62}{2695}$,
e Assuming the question means only two of the numbers are greater than 50 the probability is $\frac{75}{198}$. If the question means at least two, then the probability is $\frac{1}{2}$.

Exercise 9.4

1 **a** 22 100, **b** **i** $\frac{1}{52}$, **ii** $\frac{11}{850}$ **2** **a** $\frac{1}{6}$, **b** \$8.33, **c** no **3** **b** game 3, **c** yes, game 3

5 **a** 2 598 960, **b** **i** $\frac{1}{13}$, **ii** $\frac{33}{66\,640}$ **6** **a** no, **b** games 2 and 3, **c** no, the expected return is \$8.33 for each game

7 $P(\text{Jae wins}) = \frac{1}{2}$, $P(\text{Jemima wins}) = \frac{1}{3}$, $P(\text{Draw}) = \frac{1}{6}$. The game is not fair.

8 $P(\text{Blake wins}) = \frac{1}{3}$, $P(\text{Cynthia wins}) = \frac{1}{3}$, $P(\text{Draw}) = \frac{1}{3}$. The game is fair.

9 **a** $8^6 = 262\,144$, **b** **i** $\frac{1}{8}$, **ii** $\frac{1}{512}$, **iii** $\frac{729}{4096}$, **iv** $\frac{1}{64}$ **10** **a** **i** $\frac{2}{5}$, **ii** $\frac{2}{5}$, **b** **i** $\frac{3}{5}$, **ii** $\frac{1}{5}$, **c** **i** $\frac{1}{3}$, **ii** $\frac{1}{3}$

11 Limb = \$15, Eyesight = \$50, Hearing = \$10, Death = \$1.

15 $P(\text{Ngaire wins}) = \frac{1}{2}$, $P(\text{Mairaid wins}) = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots < \frac{1}{2}$. The game is not fair.

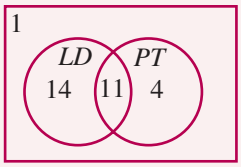
16 $P(\text{Jae wins}) = \frac{3}{5}$, $P(\text{Jemima wins}) = \frac{2}{5}$. The game is not fair.

ANSWERS

Chapter 9 Review

1 **a** J and L, **b** the card is not a black Jack, **c** **i** $\frac{1}{2}$, **ii** $\frac{1}{2}$, **iii** $\frac{6}{13}$ **2** **a** $\frac{5}{36}$, **b** $\frac{1}{2}$, **c** $\frac{5}{12}$ **3** **a** $\frac{9}{19}$, **b** $\frac{1}{19}$, **c** $\frac{1}{38}$, **d** $\frac{9}{19}$, **e** $\frac{4}{19}$ **4** **a** **i** $\frac{2317}{33\,000}$, **ii** $\frac{813}{40\,000}$, **iii** $\frac{2477}{57\,000}$, **b** Utility, **c** $\frac{1959}{59\,500}$ **5** **a** 100, **b** **i** $\frac{6}{25}$, **ii** $\frac{7}{10}$, **c** $\frac{3}{10}$, **d** $\frac{3}{8}$

6



a **i** $\frac{1}{6}$, **ii** $\frac{11}{30}$, **iii** $\frac{1}{30}$, **b** $\frac{11}{15}$ **7** **a** 12, **b** **i** $\frac{1}{4}$, **ii** $\frac{1}{12}$, **iii** $\frac{1}{3}$, **iv** $\frac{1}{2}$, **v** $\frac{1}{2}$

8 **a** AB, AT, AC, BB, BT, BC, CB, CT, CC, DB, DT, DC, **b** **i** $\frac{1}{4}$, **ii** $\frac{1}{12}$, **iii** $\frac{1}{3}$, **iv** $\frac{1}{3}$

9 **a** HOR, HOB, HOW, HER, HEB, HEW, TOR, TOB, TOW, TER, TEB, TEW

b **i** $\frac{1}{2}$, **ii** $\frac{1}{6}$, **iii** $\frac{1}{12}$, **iv** $\frac{1}{3}$, **v** $\frac{1}{2}$ **10** **b** **i** $\frac{1}{18}$, **ii** $\frac{1}{9}$, **iii** $\frac{13}{36}$, **iv** $\frac{7}{9}$, **v** $\frac{7}{18}$

11 **a** **i** $\frac{12}{25}$, **ii** $\frac{1}{4}$, **iii** $\frac{27}{100}$, **b** **i** $\frac{31}{50}$, **ii** $\frac{29}{100}$, **iii** $\frac{9}{100}$, **c** best estimate is 12 white, 6 yellow, 2 black,

d best estimate is red — 173°, blue — 90°, green — 97° **13** **a** **i** $\frac{9}{19}$, **ii** $\frac{81}{361}$, **iii** $\frac{729}{6859}$, **iv** $\frac{3\,486\,784\,401}{6\,131\,066\,257\,801}$,

b **i** $\left(\frac{9}{19}\right)^{26}$ **14** **a** WX, WY, WZ, XW, XY, XZ, YW, YX, YZ, ZW, ZX, ZY, **b** **i** $\frac{1}{6}$, **ii** $\frac{1}{2}$, **iii** $\frac{1}{12}$, **iv** $\frac{1}{2}$

15 **a**

VICE-CAPTAIN

CAPTAIN	A	B	C	D	E	F	G
A	—	AB	AC	AD	AE	AF	AG
B	BA	—	BC	BD	BE	BF	BG
C	CA	CB	—	CD	CE	CF	CG
D	DA	DB	DC	—	DE	DF	DG
E	EA	EB	EC	ED	—	EF	EG
F	FA	FB	FC	FD	FE	—	FG
G	GA	GB	GC	GD	GE	GF	—

b **i** $\frac{2}{7}$, **ii** $\frac{1}{21}$, **iii** $\frac{5}{7}$, **iv** $\frac{1}{42}$, **v** $\frac{5}{42}$, **vi** $\frac{2}{21}$

16 **a** CA, CM, CJ, CH, AM, AJ, AH, MJ, MH, JH

b **i** $\frac{2}{5}$, **ii** $\frac{3}{10}$, **iii** $\frac{3}{10}$ **17** $\frac{1}{10}$

18 **a** $\frac{1}{110}$, **b** $\frac{1}{55}$, **c** $\frac{1}{55}$, **d** $\frac{53}{55}$, **e** $\frac{28}{55}$

19 **a** $\frac{1}{90}$, **b** $\frac{1}{45}$, **c** $\frac{28}{45}$, **d** $\frac{8}{45}$, **e** $\frac{1}{30}$

20 **a** 2 598 960, **b** 308 915 776, **c** 9 765 625

21 $P(\text{Jemima wins}) = \frac{1}{4}$, $P(\text{Sam wins}) = \frac{1}{4}$, the game is fair **22** **a** \$8.33, not fair, **b** \$8.33, not fair, **c** **i** $\frac{1}{2}$, **ii** \$10, **iii** yes **23** $P(\text{David wins}) = \frac{1}{2}$, $P(\text{Samantha wins}) = \frac{1}{2}$, the game is fair

10 Further trigonometry

Exercise 10.1

1 **a** 0.64, **b** 0.34, **c** 0.87, **d** 0.98 **2** $\sin(180° - \theta) = \sin\theta$ **3** **a** $\cos 50° = 0.64$, $\cos 130° = -0.64$, **b** $\cos 10° = 0.98$, $\cos 170° = -0.98$, **c** $\cos 40° = 0.77$, $\cos 140° = -0.77$, **d** $\cos 80° = 0.17$, $\cos 100° = -0.17$ **4** $\cos(180° - \theta) = -\cos\theta$ **5** **a** $\sin 150°$, **b** $\sin 130°$, **c** $\sin 45°$, **d** $\sin 75°$, **e** $\sin 107°$, **f** $\sin 68°$, **g** $\sin 81°$, **h** $\sin 13°$ **6** **a** $\cos 160°$, **b** $\cos 110°$, **c** $\cos 60°$, **d** $\cos 25°$, **e** $\cos 95°$, **f** $\cos 78°$, **g** $\cos 139°$, **h** $\cos 43°$ **7** 1.20 **8** **a** 30°, 150°, **b** 53°, 127°, **c** 12°, 168°, **d** 34°, 146°, **e** 8°, 172°, **f** 48°, 132°, **g** 21°, 159°, **h** 55°, 125°, **i** 29°, 151° **9** **a** 60°, **b** 70°, **c** 160°, **d** 140°, **e** 65°, **f** 106°, **g** 32°, **h** 76°, **i** 123° **10** **a** 0.8, **b** 0.94, **c** 0.96, **d** 0.54, **e** 0.44, **f** 0.77, **g** 0.92, **h** 0.98

Exercise 10.2

1 **a** positive, **b** positive, **c** negative, **d** negative, **e** positive, **f** positive, **g** negative, **h** positive, **i** positive, **j** positive, **k** negative, **l** negative, **m** positive, **n** positive, **o** negative, **p** positive **2** **a** acute, **b** obtuse, **c** obtuse, **d** acute, **e** obtuse, **f** acute **3** **a** $\sin 80° = 0.98$, **b** $-\cos 40° = -0.77$, **c** $-\tan 20° = -0.36$, **d** $-\cos 55° = -0.57$, **e** $-\tan 46° = -1.04$, **f** $\sin 28° = 0.47$, **g** $-\tan 79° = -5.14$, **h** $\sin 65° = 0.91$, **i** $-\cos 72° = -0.31$, **j** $\sin 16° = 0.28$, **k** $-\tan 58° = -1.60$, **l** $-\cos 9° = -0.99$, **m** $-\tan 21° = -0.38$, **n** $-\cos 27° = -0.89$, **o** $\sin 43° = 0.68$, **p** $-\cos 86° = -0.07$, **q** $\sin 84° = 0.99$, **r** $-\tan 66° = -2.25$, **s** $-\cos 48° = -0.67$, **t** $\sin 11° = 0.19$

4 **a** $-\tan 51°45' = -1.2685$, **b** $\sin 74°33' = 0.9639$, **c** $-\cos 15°52' = -0.9619$, **d** $\sin 43°40' = 0.6905$, **e** $-\cos 66°9' = -0.4043$, **f** $-\tan 22°16' = -0.4095$, **g** $-\cos 30°24' = -0.8625$, **h** $-\tan 80°53' = -6.2316$, **i** $\sin 14°28' = 0.2498$ **5** **a** 9°, 171°, **b** 15°, 165°, **c** 57°, 123°, **d** 37°, 143°, **e** 4°, 176°, **f** 25°, 155°, **g** 11°, 169°, **h** 49°, 131°, **i** 23°, 157° **6** **a** 121°, **b** 173°, **c** 101°, **d** 137°, **e** 118°, **f** 147°, **g** 106°, **h** 134°, **i** 112°

7 **a** 32°, 148°, **b** 36°, **c** 129°, **d** 117°, **e** 62°, **f** 11°, 169°, **g** 146°, **h** 63°, 117, **i** 65°, **j** 48°, 132°, **k** 104°, **l** 127°

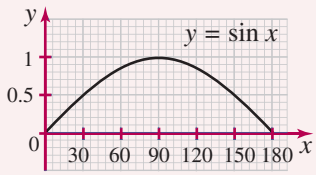
8 **a** $\frac{1}{2}$, **b** $-\frac{1}{\sqrt{2}}$, **c** $-\sqrt{3}$, **d** $-\frac{\sqrt{3}}{2}$, **e** $\frac{\sqrt{3}}{2}$, **f** -1, **g** $\frac{1}{\sqrt{2}}$, **h** $-\frac{1}{\sqrt{3}}$, **i** $-\frac{1}{2}$

9 **a**

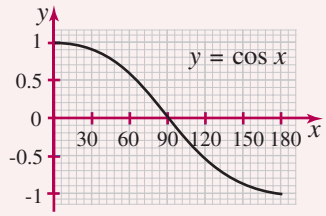
x	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
$\sin x$	0.0	0.2	0.3	0.5	0.6	0.8	0.9	0.9	1.0	1.0
$\cos x$	1.0	1.0	0.9	0.9	0.8	0.6	0.5	0.3	0.2	0.0
$\tan x$	0.0	0.2	0.4	0.6	0.8	1.2	1.7	2.7	5.7	—

x	100°	110°	120°	130°	140°	150°	160°	170°	180°
$\sin x$	1.0	0.9	0.9	0.8	0.6	0.5	0.3	0.2	0.0
$\cos x$	−0.2	−0.3	−0.5	−0.6	−0.8	−0.9	−0.9	−1.0	−1.0
$\tan x$	−5.7	−2.7	−1.7	−1.2	−0.8	−0.6	−0.4	−0.2	0.0

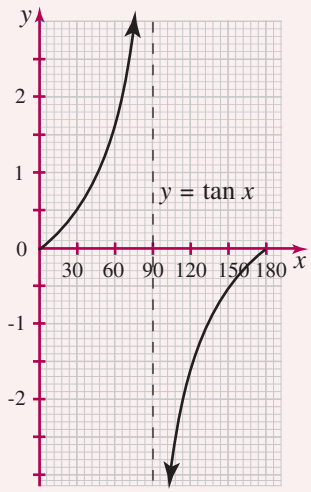
9 **b** **i**



ii



iii



10 **a** 1, **b** 0, **c** **i** $0° < x < 180°$, **ii** none, **d** $x = 0°, 180°$
11 **a** 1, **b** −1, **c** **i** $0° \leqslant x < 90°$, **ii** $90° < x \leqslant 180°$, **d** $x = 90°$
12 **a** no, **b** $x = 90°$, **c** **i** $0° < x < 90°$, **ii** $90° < x < 180°$, **d** $x = 0°, 180°$

Exercise 10.3

1 **a** $a = 10.5$, **b** $v = 22.5$, **c** $m = 32$ **2** **a** $\frac{2}{3}$, **b** 0.6, **c** $\frac{1}{12}$ **3** **a** $a = 8.9$, **b** $q = 25.2$, **c** $k = 2.7$, **d** $h = 86.5$, **e** $n = 27.3$, **f** $t = 24.9$ **4** **a** 54°, **b** 15°, **c** 28°, **d** 25°, **e** 68°, **f** 36° **5** **a** $p = 20.8$, **b** $w = 22.2$, **c** $c = 76.0$, **d** $f = 34.9$, **e** $u = 20.6$, **f** $z = 51.4$ **6** **a** 27°31′, **b** 7°5′, **c** 43°45′, **d** 29°38′, **e** 21°44′, **f** 51°30′ **7** **a** 132°, **b** 160°, **c** 120°, **d** 128°, **e** 114°, **f** 148° **8** **b** 71°, **c** 141.4 cm **9** **a** 102 mm, **b** 76 mm, **c** 208 mm **10** **a** 20 cm, **b** $6\sqrt{6}$ cm, **c** 24 cm **11** **a** 46°, 134°, **b** 29°, 151°, **c** 16°, 164° **12** **b** 24°14′, 155°46′, **c** 139°18′, 7°46′ **13** 52°

Exercise 10.4

1 **a** $a = 10.4$, **b** $u = 70.7$, **c** 54.3°, **d** 22°7′ **2** 17.7 m **3** 67 cm **4** **a** 34°, **b** 26.3 km, **c** 9.3 km **5** 20.3 cm **6** 73°44′ **7** **b** 16 cm **8** **a** 12.1 cm, **b** 31°22′ **9** **a** $LM = 76.2$ m, $LN = 93.3$ m, **b** \$12 558 **10** 54 cm **11** 37.2 cm **12** Betty is closer by 6 m **13** **b** 45° **14** **a** 12.2 km, **b** 73 m, **c** 38.5 m, **d** 338 m **15** **a** $\angle ACB = 39°$, bearing = 129°, **b** $\angle PRQ = 23°$, bearing = 203°, **c** $\angle FHG = 37°$, bearing = 307°, **d** $\angle YZX = 46°$, bearing = 046°

Exercise 10.5

1 **a** $g = \sqrt{11}$, **b** $z = \sqrt{13}$, **c** $p = 5$ **2** **a** $\frac{1}{8}$, **b** $\frac{5}{7}$, **c** $-\frac{1}{5}$ **3** **a** $p = 11.7$, **b** $k = 13.5$, **c** $v = 17.0$, **d** $t = 34.3$, **e** $g = 25.7$, **f** $d = 79.8$ **4** **a** 56°, **b** 87°, **c** 102°, **d** 79°, **e** 50°, **f** 146° **5** **a** $a = 5.2$, **b** $q = 10.3$, **c** $m = 10.8$, **d** $z = 17.8$, **e** $r = 46.1$, **f** $j = 151.1$ **6** **a** 80°24′, **b** 47°26′, **c** 114°37′, **d** 62°14′, **e** 9°46′, **f** 85°49′, **g** 17°45′, **h** 98°36′, **i** 60°19′ **7** **a** $c^2 = a^2 + b^2$, **b** Pythagoras' Theorem is derived. **8** $x = 26$ **10** 24°, 31°, 125° **11** **a** $x = \sqrt{13}$, **b** $x = \sqrt{21}$, **c** $x = \sqrt{73}$ **12** **a** 45°, **b** 120°, **c** 30°

ANSWERS

Exercise 10.6

1 **a** 10.4 cm, **b** 11.97 m, **c** 41.75°, **d** 128°12′ **2** **a** 76°32′, **b** 40°25 **3** 7° **4** 817 m **5** 63.7 km
6 $QS = 29.4$ cm, $PR = 17.4$ cm **7** 70°18′ **8** **a** Amelia — 240 km, Elizabeth — 210 km, **b** 399 km
9 **a** 93°40′, **b** 86°20′, **c** 11.6 cm **10** **a** 13 cm, **b** 131°29′ **11** **b** 120° **12** **a** 82.8 km, **b** 29.1 km, **c** 12.5 km, **d** 12.6 km **13** **a** $\angle ABC = 77°$, bearing = 035°, **b** $\angle PQR = 113°$, bearing = 239°, **c** $\angle XYZ = 96°$, bearing = 158°, **d** $\angle EFG = 82°$, bearing = 316°

Exercise 10.7

1 **a** 64.0 cm^2, **b** 185.5 cm^2, **c** 331.6 cm^2, **d** 353.1 cm^2, **e** 90.3 cm^2, **f** 244.1 cm^2 **2** **a** 55.41 mm^2, **b** 220.3 cm^2, **c** 625 m^2 **3** **a** 239.6 cm^2, **b** 95.7 cm^2, **c** 888.3 cm^2 **4** **a** 120 cm^2, **b** 5 cm **5** 62 cm^2 **6** 253 cm^2
7 **a** $16\sqrt{3}$ cm^2, **b** 12.5 cm^2, **c** 21 cm^2 **8** **a** 42°, **b** 24°, **c** 152° **9** **a** 1973 cm^3, **b** 153.6 cm^3 **10** 5.35 ha
11 **a** $A = \frac{1}{2}ab$, **b** The standard formula for the area of a triangle is derived. **12** **a** 136.8 cm^2, **b** 96.5 cm^2, **c** 40.3 cm^2 **13** $24\sqrt{3}$ cm^2 **14** $h = 4.5$

Exercise 10.8

1 **a** $x = 10.3$, **b** $k = 8.9$, **c** $p = 21.8$, **d** $n = 100.7$, **e** $a = 17.1$, **f** $z = 113.9$ **2** **a** 33°35′, **b** 71°47′, **c** 97°11′, **d** 33°2′, **e** 169°17′, **f** 48°50′ **3** 40° **4** 150° **5** **a** 25.61 cm, **b** 38°32′ **6** 332 m **7** 105°24′
8 20 m **9** **a** 83°31′, **b** 171 mm^2 **10** **a** 153°, **b** 399 m^2 **11** **a** 114°, **b** 8 cm **12** **b** 52.6 cm
13 **b** $4\sqrt{6}$ cm **14** 41 m^2 **15** **a** **i** 62°11′, **ii** 19.9 m^2, **b** **i** 84°16′, **ii** 19.9 m^2 **16** **a** 21 m, **b** 142.5 m^2
18 **a** $x = 11$, **b** $y = \sqrt{97}$ **19** **a** 18 km, **b** 050° **20** 1008 NM **21** **a** 77°24′, **b** 18.9 cm **22** **a** 56.6 cm, **b** 28° **23** **a** 125°, **b** 73° **24** **a** 26.7 cm, **b** 19.4 cm **25** **a** 22 cm, **b** 13 cm **26** **a** 58°, **b** 49°, **c** 13.6 cm
27 **a** 16.54 km, **b** 13 900 m

Chapter 10 Review

1 **a** acute, **b** obtuse **2** **a** 0.94, **b** –0.64, **c** –0.27, **d** 0.81, **e** –0.78, **f** –0.89 **3** **a** 45°, 135°, **b** 23°, 157°,
4 **a** 114°, **b** 109° **5** **a** $\frac{1}{\sqrt{2}}$, **b** $-\frac{1}{2}$, **c** $-\frac{1}{\sqrt{3}}$ **6** **a** $a = 15.4$, **b** $p = 81.2$, **c** $y = 34.5$, **d** $k = 8.0$, **e** $w = 46.1$,
f $n = 3.1$ **7** **a** 46°46′, **b** 127°38′, **c** 75°36′, **d** 47°13′, **e** 99°43′, **f** 41°13′ **8** 562 m **9** 71°47′
10 **b** $\angle UTV = 35°$, $\angle TUV = 84°$, **c** 119 km **11** **b** 26°, **c** 116° **12** **b** 46°, **c** 136°, **d** 316°
13 **a** 30.7 cm^2, **b** 117.8 cm^2 **14** **a** 71°12′, **b** 143°53′ **15** 112.1 cm^2 **16** **a** 984.8 cm^3, **b** 45.8 cm^3
17 222 mm^2 **18** **a** 22 cm, **b** 48°, **c** 6.2 cm **19** **a** 22.5 cm, **b** 76°28′

11 Similarity

Exercise 11.1

1 **a** $AB = 10$ mm, $BC = 25$ mm, $CD = 30$ mm, $DA = 15$ mm; $PQ = 20$ mm, $QR = 50$ mm, $RS = 60$ mm, $SP = 30$ mm **b** **i** 1 : 2, **ii** 1 : 2, **iii** 1 : 2, **iv** 1 : 2, **c** They are equal. **d** 2 **2** **a** $\angle X = 45°$, $\angle Y = 108°$, $\angle Z = 27°$; $\angle L = 45°$, $\angle M = 108°$, $\angle N = 27°$, **b** Yes **3** **a** AB and PQ, BC and QR, CD and RS, DA and SP, **b** KL and FG, LM and GH, MN and HE, NK and EF. **4** **a** $\angle X$ and $\angle D$, $\angle Y$ and $\angle E$, $\angle Z$ and $\angle C$, **b** $\angle S$ and $\angle G$, $\angle T$ and $\angle H$, $\angle U$ and $\angle I$, $\angle V$ and $\angle J$, $\angle W$ and $\angle K$ **5** **a** yes, **b** no, **c** yes, **d** no, **e** no, **f** no **6** **a** $\frac{5}{3}$, **b** $\frac{12}{7}$, **c** $\frac{8}{5}$, **d** $\frac{9}{4}$
7 **a** 2, $x = 8$, **b** 3, $p = 12$, **c** 5, $a = 45$, **d** $\frac{3}{2}$, $t = 15$, **e** $\frac{1}{2}$, $c = 7$, **f** $\frac{1}{4}$, $e = 5$ **8** **a** yes, **b** yes
9 **a** $\frac{2}{3}$, **b** $\frac{10}{7}$ **10** **a** $\frac{2}{5}$, **b** $\frac{4}{3}$ **11** **a** yes, **b** no **12** **a** 1 : 4, 1 : 4, yes, **b** 4 : 7, 4 : 5, no, **c** 2 : 5, 2 : 5, yes,
d 2 : 3, 3 : 4, no **13** **a** C and D, **b** A and C, **c** B and D **14** **a** $\frac{PQ}{TQ} = \frac{QR}{QV} = \frac{RS}{VU} = \frac{SP}{UT} = \frac{4}{3}$, $\angle P = \angle T$, $\angle Q$ is common, $\angle R = \angle V$, $\angle S = \angle U$, $\therefore PQRS \,|||\, TQVU$ (matching sides are in proportion and matching angles are equal). **b** $\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GD} = \frac{DA}{DE} = 2$, $\angle A = \angle E$, $\angle B = \angle F$, $\angle C = \angle G$, $\angle D$ is common, $\therefore ABCD \,|||\, EFGD$ (matching sides are in proportion and matching angles are equal).

Exercise 11.2

1 **a** EF and YX, FG and XZ, EG and YZ, **b** LM and DE, MN and CD, LN and CE **2** **a** $\angle K$ and $\angle U$, $\angle L$ and $\angle V$, $\angle M$ and $\angle T$, **b** $\angle C$ and $\angle J$, $\angle D$ and $\angle K$, $\angle E$ and $\angle I$ **3** **a** $\alpha = 60°$, $\beta = 70°$, yes **b** $\alpha = 48°$, $\beta = 32°$, no **c** $\alpha = 30°$, $\beta = 120°$, yes **d** $\alpha = 70°$, $\beta = 63°$, yes **4** **a** $\frac{8}{4} = \frac{10}{5} = \frac{12}{6} = 2$, yes, **b** $\frac{39}{13} = 3$, $\frac{24}{8} = 3$, $\frac{36}{11} = 3\frac{3}{11}$, no, **c** $\frac{2.5}{10} = \frac{3}{12} = \frac{4.5}{18} = \frac{1}{4}$, yes, **d** $\frac{5}{21} = 0.208$, $\frac{5.2}{25} = 0.208$, $\frac{5.6}{28} = 0.2$, no **5** **a** yes, **b** no, **c** yes, **d** no **6** **a** yes, **b** no, **c** yes, **d** no **7** **a** $x = 12$, **b** $c = 27$, **c** $g = 35$, **d** $k = 36$, **e** $f = 33$, **f** $q = 35$ **8** **a** $p = 38$, $q = 77$, **b** $x = 53$, $y = 37$, **c** $e = 44$, $f = 46$ **9** **a** $m = 8$, **b** $x = 16$, **c** $g = 48$, **d** $u = 24$ **10** **a** $x = 12$, **b** $k = 3$, **c** $t = 72$, **d** $h = 18$, **e** $n = 14$, **f** $d = 36$ **11** **a** a = 8, $b = 12$, **b** $c = 8$, $d = 15$, **c** $x = 8$, $y = 22.5$, **d** $p = 12$, $q = 42$, **e** $u = 7.5$, $v = 12$, **f** $m = 16.5$, $n = 18$ **12** **a** a = 6, **b** $p = 15$, **c** $x = 21$ **13** **a** **iii** $x = 18$, $y = 30$, **b** **iii** $a = 15$, $b = 28$, **c** **iii** $e = 15$, $f = 5$

Exercise 11.3

1 1.5 m **2** 6 m **3** 21 m **4** 72 m **5** 15 m **6** 2100 m **7** 1.92 m **8** 45 cm **9** 10 m **10** 16 m **11** 3 cm

Exercise 11.4

1 **a** $\angle BAC = \angle CED$ (alternate $\angle$s, $AB \parallel DE$)
$\angle ACB = \angle DCE$ (vertically opposite $\angle$s)
$\therefore \Delta ABC \,|||\, \Delta EDC$ (equiangular); $x = 15$, $y = 12$

c $\frac{XY}{UV} = \frac{8}{20} = \frac{2}{5}$
$\frac{XZ}{VW} = \frac{10}{25} = \frac{2}{5}$
$\frac{YZ}{UW} = \frac{14}{35} = \frac{2}{5}$
$\therefore \Delta XYZ \,|||\, \Delta VUW$ (matching sides in proportion); $a = 44$

e $\frac{DE}{EG} = \frac{10}{15} = \frac{2}{3}$
$\frac{DE}{EF} = \frac{12}{18} = \frac{2}{3}$
$\frac{EF}{FG} = \frac{18}{27} = \frac{2}{3}$
$\therefore \Delta DEF \,|||\, \Delta EGF$ (matching sides in proportion); $g = 110$

g $\angle VWZ = \angle XYZ = 90°$ (given)
$\angle Z$ is a common angle
$\therefore \Delta VWZ \,|||\, \Delta XYZ$ (equiangular); $x = 40$

i $\frac{FG}{JI} = \frac{10}{16} = \frac{5}{8}$
$\frac{GH}{FJ} = \frac{25}{40} = \frac{5}{8}$
$\frac{FH}{FI} = \frac{30}{48} = \frac{5}{8}$
$\therefore \Delta FGH \,|||\, \Delta IJF$ (matching sides are in proportion); $u = 110$

b $\frac{QR}{RT} = \frac{6}{8} = \frac{3}{4}$
$\frac{PR}{RS} = \frac{9}{12} = \frac{3}{4}$
$\angle QRP = \angle TRS$ (vertically opposite $\angle$s)
$\therefore \Delta PQR \,|||\, \Delta STR$ (sides about equal $\angle$s are in proportion); $m = 16$

d $\angle JMN = \angle JKL$ (corresponding $\angle$s, $MN \parallel KL$)
$\angle J$ is a common angle
$\therefore \Delta JMN \parallel \Delta JKL$ (equiangular); $k = 17.5$

f $\frac{TU}{RS} = \frac{12}{21} = \frac{4}{7}$
$\frac{TV}{ST} = \frac{20}{35} = \frac{4}{7}$
$\frac{UV}{RT} = \frac{24}{42} = \frac{4}{7}$
$\therefore \Delta TUV \,|||\, \Delta SRT$ (matching sides are in proportion); $r = 56$

h $\frac{PS}{PQ} = \frac{15}{18} = \frac{5}{6}$
$\frac{PT}{PR} = \frac{20}{24} = \frac{5}{6}$
$\angle P$ is a common angle
$\therefore \Delta PST \,|||\, \Delta PQR$ (sides about equal $\angle$s are in proportion); $e = 30$

2 **a** $\angle APQ = \angle ABC$ (corresponding $\angle$s, $PQ \parallel BC$)
$\angle A$ is a common angle
$\therefore \Delta APQ \,|||\, \Delta ABC$ (equiangular)

b $PQ = 15$, $AQ = 12.5$, $QC = 10$

ANSWERS

3 a $\frac{QT}{TS} = \frac{7.2}{9} = \frac{4}{5}$

$\frac{RT}{TP} = \frac{4.8}{6} = \frac{4}{5}$

$\angle QTR = \angle PTS$ (vertically opposite $\angle$s)

$\therefore \Delta QTR \,|||\, \Delta STP$ (sides about equal $\angle$s are in proportion)

b $\angle RQT = \angle PST$ (matching $\angle$s in similar Δs)

$\therefore QR \parallel PS$ (alternate $\angle$s are equal)

$\therefore PQRS$ is a trapezium (one pair of opposite sides are parallel)

4 a $\angle EDF = \angle EAG$ (corresponding $\angle$s, $DC \parallel AB$)

$\angle E$ is a common angle

$\therefore \Delta DEF \,|||\, \Delta AEG$ (equiangular)

b $AG = 9$

5 a $\frac{VW}{XW} = \frac{3}{4}$

$\frac{WY}{UW} = \frac{6}{8} = \frac{3}{4}$

$\angle W$ is a common angle

$\therefore \Delta WUX \,|||\, \Delta WYV$ (sides about equal $\angle$s are in proportion)

b $VY = 5\frac{1}{4}$

6 a $\angle MLN = \angle MYX$ (given)

$\angle M$ is a common angle

$\therefore \Delta LMN \,|||\, \Delta YMX$ (equiangular)

b $MN = 28$, $YN = 4$

7 a $\frac{PQ}{QR} = \frac{6}{12} = \frac{1}{2}$

$\frac{QS}{PQ} = \frac{3}{6} = \frac{1}{2}$

$\angle Q$ is a common angle

$\therefore \Delta PQS \,|||\, \Delta RQP$ (sides about equal $\angle$s are in proportion)

b $\angle PSQ = \angle RPQ$ (matching $\angle$s in similar Δs)

8 a $\angle ABC = \angle CDE = 90°$ (given)

$\angle ACB = \angle DCE$ (vertically opposite $\angle$s)

$\therefore \Delta ABC \,|||\, \Delta EDC$ (equiangular)

b $BC = 10$, $CD = 8$

9 a $\angle KML = \angle LNM$ (given)

$\angle L$ is a common angle

$\therefore \Delta KLM \,|||\, \Delta MLN$ (equiangular)

b $LN = 25$, $KN = 9$

10 $\angle CDE = \angle CAB$ (corresponding $\angle$s, $DE \parallel AB$

$\angle C$ is a common angle

$\therefore \Delta CDE \,|||\, \Delta CAB$ (equiangular)

$\frac{CE}{CB} = \frac{CD}{CA}$ (matching sides are in proportion)

$\therefore \frac{CE}{CB} = \frac{1}{2}$

$\therefore CB = 2CE$

$\therefore E$ is the midpoint of CB

$\therefore CE = EB$

11 a $\frac{PA}{PQ} = \frac{1}{2}$ (A is the midpoint of PQ)

$\frac{PB}{PR} = \frac{1}{2}$ (B is the midpoint of PR)

$\angle P$ is a common angle

$\therefore \Delta APB \,|||\, \Delta QPR$ (sides about equal $\angle$s are in proportion)

b $\frac{AB}{QR} = \frac{PA}{PQ}$ (matching sides are in proportion)

$\therefore \frac{AB}{QR} = \frac{1}{2}$

$\therefore QR = 2AB$

$\angle PAB = \angle PQR$ (matching $\angle$s in similar Δs)

$\therefore AB \parallel$ QR(corresponding $\angle$s are equal)

$\therefore QR$ is parallel to AB and twice its length

12 a Let $\angle BAD = \alpha$

$\angle ABD = 90° - \alpha$ ($\angle$ sum of ΔABD)

$\angle CBD = \alpha$ (adjacent $\angle$s in a right angle)

$\therefore \angle BAD = \angle CBD$ (both equal to α)

b $\angle BAD = \angle CBD$ (proven above)

$\angle BDA = \angle BDC = 90°$ ($BD \perp AC$)

$\therefore \Delta ABD \,|||\, \Delta BCD$ (equiangular)

c $\frac{BD}{DC} = \frac{AD}{BD}$ (matching sides are in proportion)

$\therefore BD^2 = AD.\, DC$

d $BD = 12$ cm

13 a $\angle AXY = \angle ABC$ (corresponding $\angle$s, $XZ \parallel BC$)

$\angle A$ is a common angle

$\therefore \Delta AXY \,|||\, \Delta ABC$ (equiangular)

b $\frac{AY}{AC} = \frac{AX}{AB}$ (matching sides are in proportion)

$\therefore \frac{AY}{AC} = \frac{1}{2}$ (X is midpoint of AB)

$\therefore Y$ is the midpoint of AC

c $\frac{XY}{BC} = \frac{AX}{AB}$ (matching sides are in proportion)

$\frac{XY}{10} = \frac{1}{2}$

$\therefore XY = 5$ cm

d $\angle CZY = \angle CDA$ (corresponding $\angle$s, $XY \parallel AD$)

$\angle C$ is a common angle

$\therefore \Delta CZY \,|||\, \Delta CDA$ (equiangular)

e $\frac{YZ}{AD} = \frac{CY}{CA}$ (matching sides are in proportion)

$\therefore \frac{YZ}{6} = \frac{1}{2}$ (Y is the midpoint of AC)

$\therefore YZ = 3$ cm

f $XZ = 5$ cm + 3 cm = 8 cm

14 a $\angle EHC = \angle ABC$ (corresponding $\angle$s, $AB \parallel EH$)

$\angle ACB = \angle ABC$ (base $\angle$s of isosceles Δ, $AB = AC$)

$\therefore \angle EHC = \angle ACB$ (both equal to $\angle ABC$)

$\therefore EH = EC$ (equal sides lie opposite equal $\angle$s)

b $\angle FBD = \angle EHD$ (corresponding $\angle$s, $AB \parallel EH$)

$\angle A$ is a common angle

$\therefore \Delta BFD \,|||\, \Delta HED$ (equiangular)

c $\frac{FB}{EH} = \frac{FD}{ED}$ (matching sides are in proportion)

$= \frac{2}{1}$ (E is midpoint of FD)

But, $EH = EC$ (proven above)

$\therefore \frac{FB}{EC} = \frac{2}{1}$, i.e. $FB : EC = 2 : 1$

ANSWERS

15 i Construct BD

$\frac{AW}{AB} = \frac{1}{2}$ (W is midpoint of AB)

$\frac{AZ}{AD} = \frac{1}{2}$ (Z is midpoint of AD)

$\angle A$ is a common angle

$\therefore \Delta ZAW \,|||\, \Delta DAB$ (sides about equal $\angle$s are in proportion)

ii $\frac{WZ}{BD} = \frac{AW}{AB}$ (matching sides are in proportion)

$= \frac{1}{2}$

$\therefore BD = 2WZ$

$\angle AZW = \angle ADB$ (matching $\angle$s in similar Δs)

$\therefore ZW \parallel BD$ (corresponding $\angle$s are equal)

iii $\frac{CX}{CB} = \frac{1}{2}$ (X is midpoint of BC)

$\frac{CY}{CD} = \frac{1}{2}$ (Y is midpoint of CD)

$\angle C$ is a common angle

$\therefore \Delta CXY \,|||\, \Delta CBD$ (sides about equal $\angle$s are in proportion)

iv $\frac{XY}{BD} = \frac{CX}{CB}$ (matching sides are in proportion)

$= \frac{1}{2}$

$\therefore BD = 2XY$

$\angle CXY = \angle CBD$ (matching $\angle$s in similar Δs)

$\therefore XY \parallel BD$ (corresponding $\angle$s are equal)

v $2WZ = BD$ and $2XY = BD$, $\therefore WZ = XY$

$WZ \parallel BD$ and $XY \parallel BD$, $\therefore WZ \parallel XY$

$\therefore WXYZ$ is a parallelogram (one pair of opposite sides are equal and parallel)

16 i $\angle PMY = \angle YKN$ (alternate $\angle$s, $LM \parallel KN$)

$\angle PYM = \angle KYN$ (vertically opposite $\angle$s)

$\therefore \Delta PMY \,|||\, \Delta NKY$ (equiangular)

ii $\frac{PM}{LM} = \frac{1}{2}$ (P is midpoint of LM)

$LM = KN$ (opposite sides of a parallelogram)

$\therefore \frac{PM}{KN} = \frac{1}{2}$

$\frac{MY}{YK} = \frac{PM}{KN}$ (matching sides are in proportion)

$\therefore \frac{MY}{YK} = \frac{1}{2}$

iii $\angle XKQ = \angle LMK$ (alternate $\angle$s, $LM \parallel KN$)

$\angle KXQ = \angle LXM$ (vertically opposite $\angle$s)

$\therefore \Delta XKQ \,|||\, \Delta XML$ (equiangular)

iv $\frac{KQ}{KN} = \frac{1}{2}$ (Q is midpoint of KN)

$LM = KN$ (opposite sides of a parallelogram)

$\therefore \frac{KQ}{LM} = \frac{1}{2}$

$\frac{KX}{XM} = \frac{KQ}{LM}$ (matching sides are in proportion)

$\therefore \frac{KX}{XM} = \frac{1}{2}$

v $\frac{MY}{YK} = \frac{KX}{XM} = \frac{1}{2}$, $\therefore KX = XY = YM$

$\therefore LQ$ and PN trisect KM at X and Y

Exercise 11.5

1 a 25 : 9, **b** 1 : 4, **c** 4 : 9, **d** 16 : 9, **e** 49 : 100, **f** 9 : 4 **2 a** 36 : 25, **b** 3 : 4, **c** 81 : 25, **d** 10 : 13 **3 a** 50 cm^2, **b** 196 cm^2, **c** 52 cm^2, **d** 120 cm^2, **e** 165.6 cm^2, **f** 100 cm^2 **4 a** $p = 16$, **b** $d = 6$, **c** $z = 13.5$, **d** $n = 30$, **e** $t = 25.2$, **f** $k = 19.5$ **5 a** 125π cm^2, **b** 10.5 cm **6** 1176 cm^2 **7 a** 9 : 13, **b** no **8** $x = 9$, $y = 20$ **9** 9 cm

Exercise 11.6

1 a i 9 : 25, **ii** 27 : 125, **b i** 4 : 1, **ii** 8 : 1, **c i** 4 : 9, **ii** 8 : 27, **d i** 25 : 16, **ii** 125 : 64, **e i** 9 : 16, **ii** 27 : 64, **f i** 49 : 25, **ii** 343 : 125 **2 a** 45 cm^2, **b** 147 cm^2, **c** 112.5 cm^2, **d** 160 cm^2 **3** no **4 a** 1625 cm^3, **b** 99.9 cm^3, **c** 825 cm^3, **d** 1996.5 cm^3 **5 a** 7 : 2, **b** 343 : 8 **6** 64 : 81 **7 a** $h = 15$, **b** $w = 63$, **c** $y = 30$, **d** $c = 28$ **8** 360 cm^3 **9 a** 9 times, **b** 27 times **10** 64 **11** 21 cm **12 a** 21%, **b** 33.1%

Chapter 11 Review

1 a scale factor = 3, $p = 15$, $q = 6$, **b** scale factor = $\frac{1}{2}$, $a = 13$, $b = 22$ **2** $\frac{5}{6}$ **3 a** $t = 21$, $u = 5$, **b** $p = 9$, $q = 24$, **c** $m = 12$, $n = 15$ **4 a** yes, equiangular Δs, **b** yes, matching sides are in proportion, **c** no, **d** yes, sides about equal $\angle$s are in proportion, **e** yes, hypotenuses and another pair of matching sides in 2 right-angled Δs are in proportion, **f** no

5 a $\angle EGF = \angle HGI$ (vertically opposite $\angle$s)
$\angle EFG = \angle GHI$ (alternate $\angle$s, $EF \parallel HI$)
$\therefore \Delta EFG \,|||\, \Delta IHG$ (equiangular)
$p = 31.5$, $q = 18$

b $\frac{ST}{TQ} = \frac{9}{6} = \frac{3}{2}$
$\angle PTS = \angle RTQ$ (vertically opposite $\angle$s)
$\frac{PT}{TR} = \frac{15}{10} = \frac{3}{2}$
$\therefore \Delta PTS \,|||\, \Delta RTQ$ (sides about equal $\angle$s are in proportion)
$c = 18$, $d = 56$

c $\frac{AD}{BD} = \frac{12}{18} = \frac{2}{3}$
$\frac{AB}{CD} = \frac{14}{21} = \frac{2}{3}$
$\frac{BD}{BC} = \frac{18}{27} = \frac{2}{3}$
$\therefore \Delta ABD \,|||\, \Delta DCB$ (matching sides are in proportion)
$m = 42$, $n = 51$

d $\frac{PY}{XY} = \frac{4}{8} = \frac{1}{2}$
$\frac{PQ}{XZ} = \frac{5}{10} = \frac{1}{2}$
$\angle PYQ = \angle XYZ = 90°$ (common angle)
$\therefore \Delta PYQ \,|||\, \Delta XYZ$ (hypotenuses and another pair of matching sides in 2 right-angled Δs are in proportion)
$u = 53$, $v = 37$

e $\angle ADE = \angle ABC$ (corresponding $\angle$s, $DE \parallel BC$)
$\angle A$ is a common angle
$\therefore \Delta ADE \,|||\, \Delta ABC$ (equiangular)
$x = 14$, $y = 5$

f $\angle XYZ = \angle XVW$ (corresponding $\angle$s, $YZ \parallel VW$)
$\angle X$ is a common angle
$\therefore \Delta XYZ \,|||\, \Delta XVW$ (equiangular)
$u = 24$, $v = 30$

g $\frac{HG}{EF} = \frac{10}{15} = \frac{2}{3}$
$\frac{FG}{ED} = \frac{14}{21} = \frac{2}{3}$
$\frac{FH}{DF} = \frac{16}{24} = \frac{2}{3}$
$\therefore \Delta FGH \,|||\, \Delta DEF$ (matching sides are in proportion)
$c = 82$, $d = 38$

h $\angle PST = \angle PRQ$ (given)
$\angle P$ is a common angle
$\therefore \Delta PST \,|||\, \Delta PRQ$ (equiangular)
$m = 23$, $n = 10$

6 12 cm

7 a $\angle PST = \angle PQR$ (corresponding $\angle$s, $ST \parallel QR$)
$\angle P$ is a common angle
$\therefore \Delta PST \,|||\, \Delta PQR$ (equiangular)

b $\frac{PT}{PR} = \frac{PS}{PQ}$ (matching sides are in proportion)
$\therefore \frac{PT}{PR} = \frac{1}{2}$
$\therefore PR = 2PT$
$\therefore PT = TR$
$\therefore T$ is the midpoint of PR

8 a Let $\angle KLN = \alpha$
$\angle LKN = 90° - \alpha$ ($\angle$ sum of ΔKLN)
$\angle MLN = 90° - \alpha$ (adjacent $\angle$s in a right angle)
$\angle LMN = \alpha$ ($\angle$ sum of ΔLMN)
Now, $\angle KLN = \angle LMN$ (both equal to α)
and $\angle LKN = \angle MLN$ (both equal to $90° - \alpha$)
$\therefore \Delta KLN \,|||\, \Delta LMN$ (equiangular)

b $\frac{LN}{NM} = \frac{KN}{LN}$ (matching sides are in proportion)
$\therefore LN^2 = KN.\, NM$

c 15 cm

9 a 16 : 49, **b** 245 cm^2 **10 a** 5 : 3, **b** 27 cm **11 a** 288 cm^2, **b** 500 cm^3 **12** 64 : 27

12 Functions and logarithms

Exercise 12.1

1 a yes, **b** yes, **c** no, **d** yes, **e** no, **f** yes **2** B **3 a** yes, **b** yes, **c** no, **d** yes **4 a** no, **b** yes, **c** yes, **d** no, **e** yes, **f** yes, **g** yes, **h** no, **i** no, **j** yes, **k** no, **l** yes **5 a** the line is not vertical, **b** the line is vertical
6 a yes, **b** no, **c** yes, **d** yes, **e** yes, **f** no, **g** yes, **h** yes **7 a** all real x; all real y, **b** all real x; $y \geqslant -9$,

ANSWERS

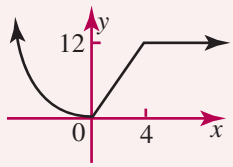
c $x \geqslant -9$; all real y, **d** $x \geqslant 0$; $y \geqslant 5$, **e** all real x; $y > 1$, **f** all real x; $-1 < y < 1$, **g** $-3 \leqslant x \leqslant 3$; $0 \leqslant y \leqslant 3$,
h all real x; $y < 8$, **i** $x > -4$; all real y, **j** all real x, $x \neq 2$; all real y, $y \neq 0$, **k** all real x, $x \neq -1$; all real y, $y \neq 1$,
l $-6 \leqslant x \leqslant 6$; $-4 \leqslant y \leqslant 4$, **m** $x \leqslant -6$ or $x \geqslant 6$; $y \geqslant 0$, **n** $-8 \leqslant x \leqslant 12$; $-9 \leqslant y \leqslant 11$, **o** all real x; $-2 \leqslant y < 3$
8 **a** $-3 \leqslant x \leqslant 3$; $-3 \leqslant y \leqslant 3$, **b** $4 \leqslant x \leqslant 8$; $3 \leqslant y \leqslant 7$

Exercise 12.2

1 **a** **i** 11, **ii** −7, **iii** 5, **b** **i** 17, **ii** 8, **iii** −2, **c** **i** 21, **ii** 0, **iii** −2, **d** **i** 0, **ii** 128, **iii** −16, **e** **i** −2, **ii** 46, **iii** $-1\frac{7}{16}$, **f** **i** 36, **ii** 0, **iii** $12 + 10\sqrt{3}$, **g** **i** 1, **ii** 16, **iii** $\frac{1}{8}$, **h** **i** 25, **ii** 36, **iii** $3 + 2\sqrt{2}$, **i** **i** 5, **ii** 4, **iii** $3\sqrt{2}$, **j** **i** $2\frac{1}{2}$, **ii** $2\frac{1}{2}$, **iii** $-5\frac{1}{5}$,
k **i** 0, **ii** 14, **iii** $-2\frac{1}{4}$, **l** **i** −3, **ii** −1, **iii** $-9\frac{2}{3}$ **2** −2 **3** **a** $x = 11$, **b** $x = \pm 3$, **c** $x = 3\frac{1}{2}$, **d** $x = 10, -4$, **e** $x = 7$,
f $x = -7, 3$, **g** $x = 9$, **h** $x = -4$ **4** $f(1) = f(-1) = 2\frac{1}{2}$ **5** **a** $18a - 2$, **b** $6a + 4$, **c** $3a - 2$ **6** $g(a) = g(b) = 0$

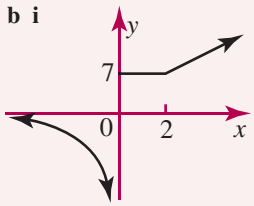
7 **a** $x + 3$, **b** $3x + 6$, **c** $x + 9$ **8** Powers of 2 are all positive (also, the graph of $y = 2^x$ never crosses the x-axis).
9 $x > -6$

12 **a** **i**



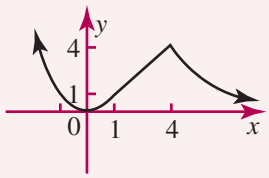
ii 27

b **i**



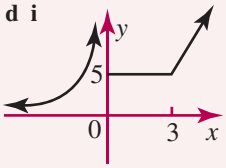
ii 14

c **i**



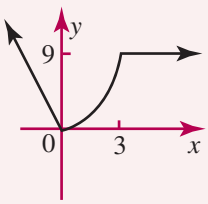
ii 5

d **i**



ii −2

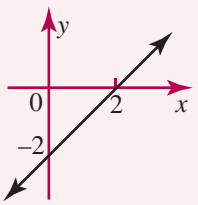
13 **a**



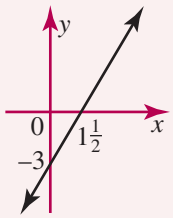
b $x = -1, 2$, **c** $x = -2\frac{1}{4}, x \geqslant 3$

Exercise 12.3

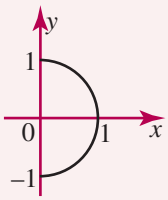
1 **a**



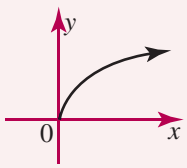
b



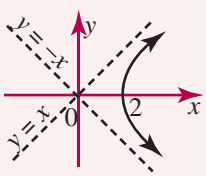
c



d



e



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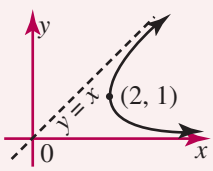

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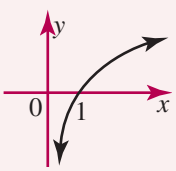
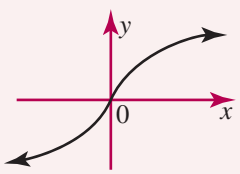
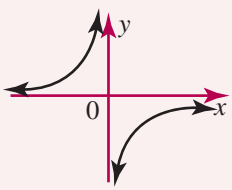


h



i



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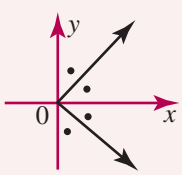

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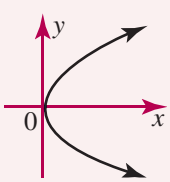
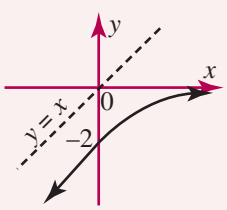
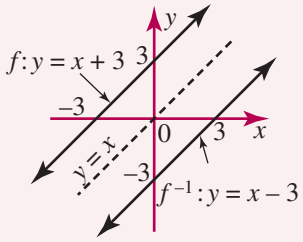


l



2 a, b, d, g, h, i, l

3 a



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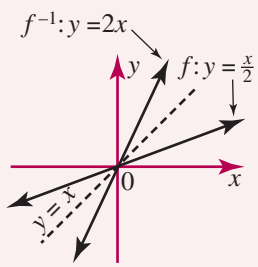

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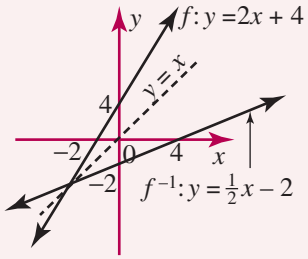

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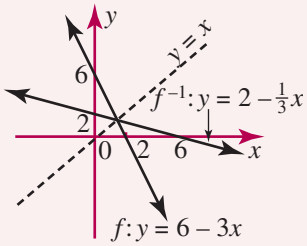

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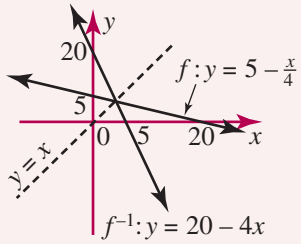

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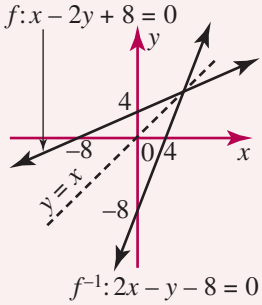
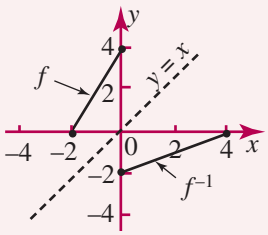
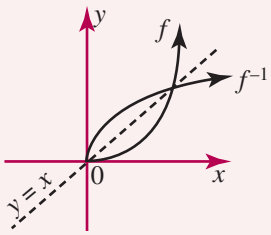


4 a $y = \frac{1}{x+2}$, **b** $y = \frac{1}{x} + 1$, **c** $y = \frac{3x+3}{1-x}$, **d** $y = \frac{5x}{x+2}$ **5 a** $y = \frac{7-2x}{x+2}$, $f(x) = f^{-1}(x)$,
b The graph is symmetric about the line $y = x$. **6** Other answers are possible. **a** $x \geqslant 0$, **b** $x \geqslant 0$, **c** $x \geqslant 3$, **d** $x \geqslant -4$, **e** $x \geqslant 1$, **f** $x \geqslant 1$ **7 a** yes, **b** no, **c** $-1 \leqslant x \leqslant 1$ **8** Because the x and y values are interchanged when finding the inverse.

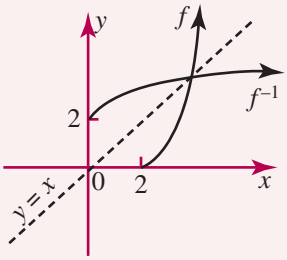
9 a i domain: $-2 \leqslant x \leqslant 0$, range: $0 \leqslant y \leqslant 4$,
ii domain: $0 \leqslant x \leqslant 4$, range: $-2 \leqslant y \leqslant 0$,
iii



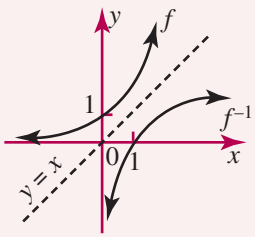
b i domain: $x \geqslant 0$, range: $y \geqslant 0$,
ii domain: $x \geqslant 0$, range: $y \geqslant 0$
iii



c i domain: $x \geqslant 2$, range: $y \geqslant 0$,
ii domain: $x \geqslant 0$, range: $y \geqslant 2$,
iii



d i domain: all real x, range: $y > 0$,
ii domain: $x > 0$, range: all real y
iii



Exercise 12.4

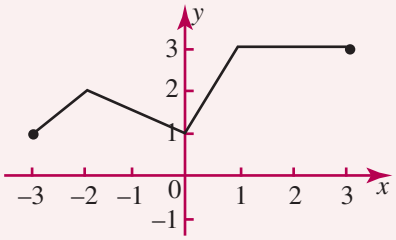
1 a $y = x^2 + 2$, **b** $y = x^2 - 3$, **c** $y = (x-1)^2$, **d** $y = (x+4)^2$ **2 a** $y = (x+2)^3$, **b** $y = x^3 + 1$, **c** $y = x^3 - 6$, **d** $y = (x-4)^3$

3 a $y = 2^x - 1$, **b** $y = 2^{x-3}$, **c** $y = 2^x + 5$, **d** $y = 2^{x+2}$ **4 a** $y = \frac{1}{x-5}$, **b** $y = \frac{1}{x} - 2$, **c** $y = \frac{1}{x+3}$, **d** $y = \frac{1}{x} + 7$

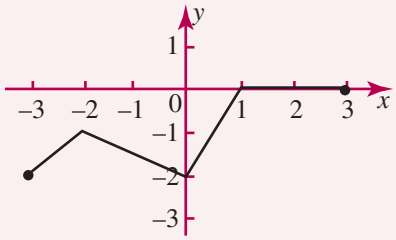
ANSWERS

5 **a** $y = x^2 + 3$, **b** $y = 3^x + 2$, **c** $y = \frac{1}{x} + 1$, **d** $y = 2^{x-3} + 1$, **e** $y = (x+1)^2 - 7$, **f** $y = \frac{1}{x-5} - 2$ **6** **a** **i** $y = 2x^2 - 5$, **ii** $y = 2(x-3)^2$, **b** **i** $y = \frac{1}{2}x^3 + 2$, **ii** $y = \frac{1}{2}(x+4)^3$, **c** **i** $y = \frac{3}{x} + 7$, **ii** $y = \frac{3}{x-1}$ **7** **a** **i** $y = -x^2 + 4$, **ii** $y = -(x-3)^2$, **b** **i** $y = 5^{-x} - 1$, **ii** $y = 5^{-(x+2)}$, **c** **i** $y = -\frac{1}{x-5}$, **ii** $y = -\frac{1}{x} - 7$

8 **a**



b



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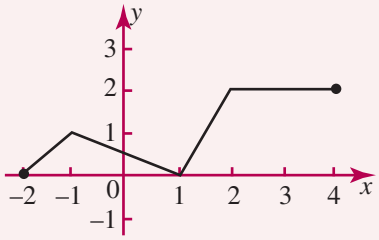

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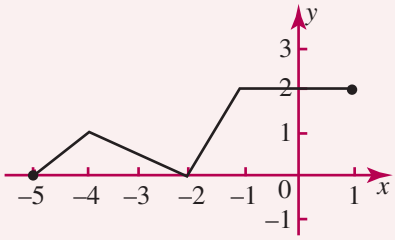

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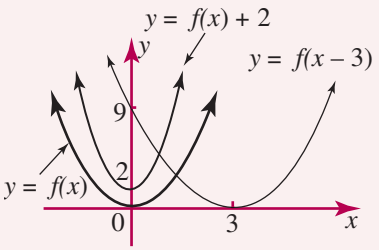

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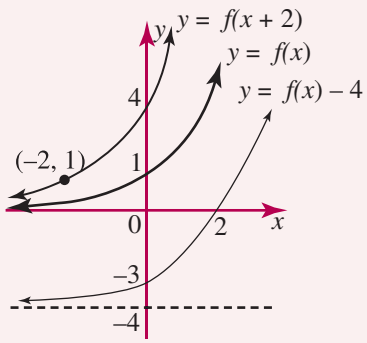
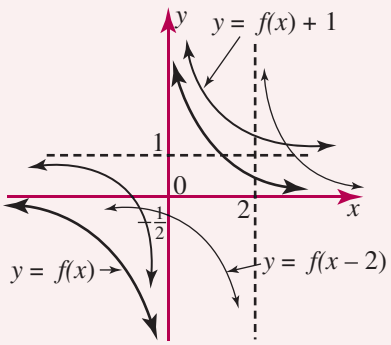


11



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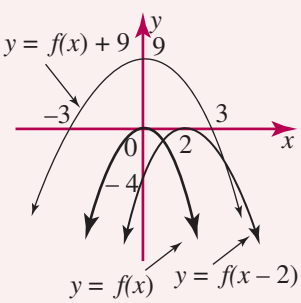

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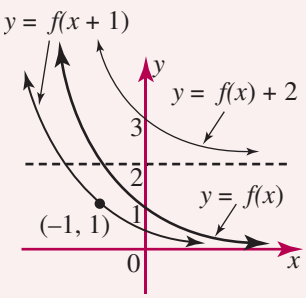

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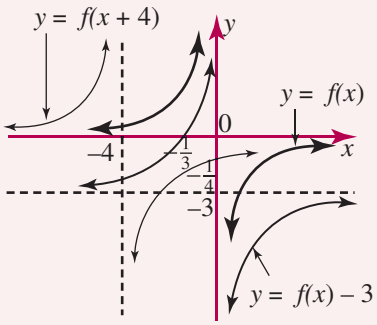
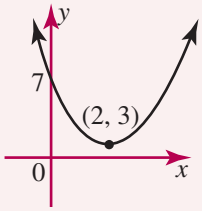


15 a



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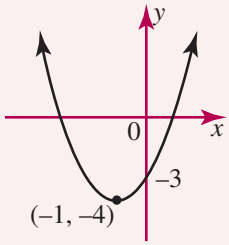

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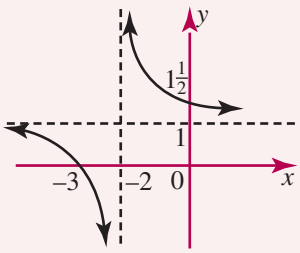
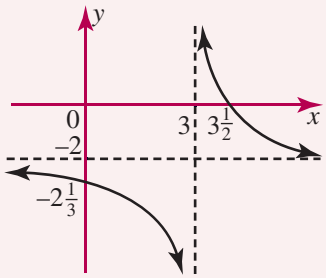
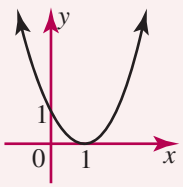


b



17 a



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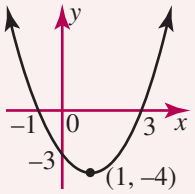

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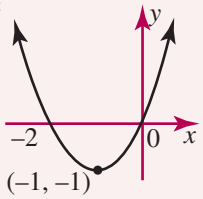
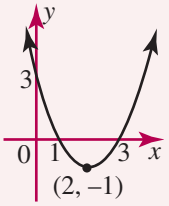
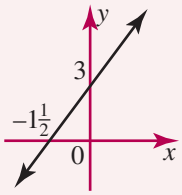


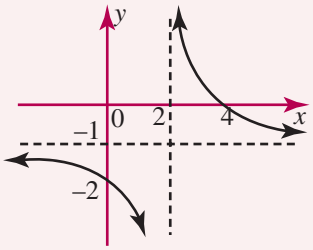
d



18 a $y = 2x + 2$, **b**



19 a $y = \frac{2}{x-1} - 1$, **b**



Exercise 12.5

1 a $x = 2$, **b** $x = 4$, **c** $x = 1$, **d** $x = 3$, **e** $x = 0$, **f** $x = 4$, **g** $x = 6$, **h** $x = 4$, **i** $x = 8$, **j** $x = 5$, **k** $x = 4$, **l** $x = 5$
2 a $x = \frac{1}{2}$, **b** $x = \frac{1}{2}$, **c** $x = \frac{1}{3}$, **d** $x = \frac{1}{4}$, **e** $x = \frac{1}{3}$, **f** $x = \frac{1}{2}$, **g** $x = \frac{1}{5}$, **h** $x = \frac{1}{4}$ **3 a** $x = -1$, **b** $x = -2$, **c** $x = -3$, **d** $x = -4$, **e** $x = -2$, **f** $x = -5$, **g** $x = -4$, **h** $x = -6$ **4 a** $x = \frac{3}{2}$, **b** $x = \frac{2}{3}$, **c** $x = \frac{4}{3}$, **d** $x = \frac{2}{3}$, **e** $x = \frac{5}{4}$, **f** $x = \frac{2}{3}$, **g** $x = \frac{5}{6}$, **h** $x = \frac{7}{5}$, **i** $x = -\frac{2}{3}$, **j** $x = -\frac{3}{2}$, **k** $x = -\frac{3}{5}$, **l** $x = -\frac{4}{3}$ **5 a** $x = 3$, **b** $x = 5$, **c** $x = -1$, **d** $x = \frac{3}{5}$, **e** $x = 4$, **f** $x = 3$, **g** $x = \frac{2}{7}$, **h** $x = -\frac{4}{7}$, **i** $x = 4$ **6 a** $x = 6$, **b** $x = \frac{5}{2}$, **c** $x = -\frac{3}{4}$, **d** $x = \frac{1}{4}$, **e** $x = -\frac{3}{2}$, **f** $x = \frac{5}{2}$, **g** $x = -\frac{13}{4}$, **h** $x = \frac{11}{4}$, **i** $x = \frac{5}{2}$ **7** $x = 2$

Exercise 12.6

1 a $\log_3 9 = 2$, **b** $\log_2 8 = 3$, **c** $\log_6 36 = 2$, **d** $\log_2 32 = 5$, **e** $\log_8 8 = 1$, **f** $\log_5 1 = 0$, **g** $\log_4 64 = 3$, **h** $\log_3 243 = 5$, **i** $\log_5(\frac{1}{5}) = -1$, **j** $\log_2(\frac{1}{4}) = -2$, **k** $\log_3 \sqrt{3} = \frac{1}{2}$, **l** $\log_7(\sqrt[3]{7}) = \frac{1}{3}$, **m** $\log_9 27 = \frac{3}{2}$, **n** $\log_2(\sqrt[5]{8}) = \frac{3}{5}$, **o** $\log_{144}(\frac{1}{12}) = -\frac{1}{2}$, **p** $\log_{16}(\frac{1}{8}) = -\frac{3}{4}$ **2 a** $5^2 = 25$, **b** $2^4 = 16$, **c** $7^1 = 7$, **d** $2^3 = 8$, **e** $3^4 = 81$, **f** $10^2 = 100$, **g** $2^5 = 32$, **h** $6^0 = 1$,

ANSWERS

i $3^5 = 243$, **j** $2^{-1} = \frac{1}{2}$, **k** $3^{-2} = \frac{1}{9}$, **l** $4^{\frac{1}{2}} = 2$, **m** $27^{\frac{1}{3}} = 3$, **n** $5^{\frac{1}{2}} = \sqrt{5}$, **o** $8^{\frac{2}{3}} = 4$, **p** $1000^{-\frac{2}{3}} = \frac{1}{100}$ **3 a** 1, **b** 2, **c** 3, **d** 5, **e** 0, **f** 3, **g** 2, **h** 4, **i** $\frac{1}{2}$, **j** $\frac{1}{3}$, **k** -1, **l** -2, **m** $-\frac{1}{2}$, **n** -2, **o** $\frac{2}{3}$, **p** $\frac{1}{4}$ **4 a** $x = 16$, **b** $x = 36$, **c** $x = 125$, **d** $x = 32$, **e** $x = 1$, **f** $x = 4$, **g** $x = 2401$, **h** $x = 1024$, **i** $x = \frac{1}{6}$, **j** $x = \frac{1}{8}$, **k** $x = \frac{1}{25}$, **l** $x = \sqrt{10}$, **m** $x = 4$, **n** $x = 8$, **o** $x = \frac{1}{7}$, **p** $x = \frac{1}{8}$ **5 a** $x = 2$, **b** $x = 3$, **c** $x = 13$, **d** $x = 2$, **e** $x = 11$, **f** $x = 4$, **g** $x = 3$, **h** $x = 10$, **i** $x = 5$, **j** $x = 6$, **k** $x = 2$, **l** $x = 6$, **m** $x = 8$, **n** $x = 8$, **o** $x = 9$, **p** $x = 32$ **6** $x = 3125$

Exercise 12.7

1 a 1, **b** 1, **c** 2, **d** 3, **e** 4, **f** 0, **g** 1, **h** 3, **i** $\frac{1}{2}$, **j** -3, **k** 2, **l** 2 **2 a** 2, **b** 2, **c** 3, **d** 3, **e** 1, **f** 3 **3 a** 2.096, **b** 0.834, **c** 1.262, **d** -0.631, **e** 0.3155, **f** 4.395, **g** 3.561, **h** 3.358, **i** 4.192, **j** -1.465, **k** -1.262, **l** 1.3635
4 a 1.079, **b** 1.255, **c** 0.301, **d** -0.176, **e** -0.125, **f** 1.556, **g** 1.431, **h** -0.602, **i** 0.389, **j** -0.2385, **k** 0.903, **l** 1.732, **m** 0.653, **n** 1.505, **o** -1.38, **p** 0.8405 **5 a** $\log(10ab)$, **b** $\log\left(\frac{4p}{3q}\right)$, **c** $\log(x^3y^2)$, **d** $\log\left(\frac{m^5}{n^4}\right)$, **e** $\log(a\sqrt{b})$, **f** $\log(p^2q)$, **g** $\log(pqr)$, **h** $\log\left(\frac{ab}{c}\right)$, **i** $\log\left(\frac{e}{fg}\right)$, **j** $\log\left(\frac{x^2z}{y^3}\right)$, **k** $\log(p^2\sqrt[3]{mn})$, **l** $\log\left(\frac{1}{y^3}\right)$
6 a $\log a + \log b + \log c$, **b** $\log p + \log q - \log r$, **c** $\log x - \log y - \log z$, **d** $\log g + \log h$, **e** $\frac{1}{3}\log p + \frac{1}{3}\log q$, **f** $-\log m - \log n$, **g** $2\log a - \frac{1}{2}\log b$, **h** $\frac{1}{2}\log c - \frac{1}{2}\log d$, **i** $\frac{2}{3}\log x - 4\log y$ **7 a** $7\log x$, **b** $5\log x$, **c** $7\log x$, **d** $7\log x$, **e** $8\log x$, **f** $4\log x$ **8 a** 2, **b** 3, **c** $\frac{2}{3}$, **d** $\frac{1}{2}$, **e** 3, **f** $\frac{3}{5}$, **g** 8, **h** 5, **i** -2 **9 a** 2.16, **b** 0.66, **c** -0.16, **d** 3.32, **e** 3.66, **f** 6.48, **g** 1.58, **h** 1.08, **i** 0.34, **j** 1.5, **k** 0.5, **l** -1.5 **10 a** $x = 7$, **b** $x = 15$, **c** $x = 7$, **d** $x = 10$, **e** $x = 24$, **f** $x = 22$ **11 a** $x = 5$, **b** $x = 4$, **c** $x = 3, 4$, **d** $x = 4, 9$, **e** $x = 4$, **f** $x = 2$ **12 a** $x = 9$, **b** $x = 5$, **c** $x = 1$

Exercise 12.8

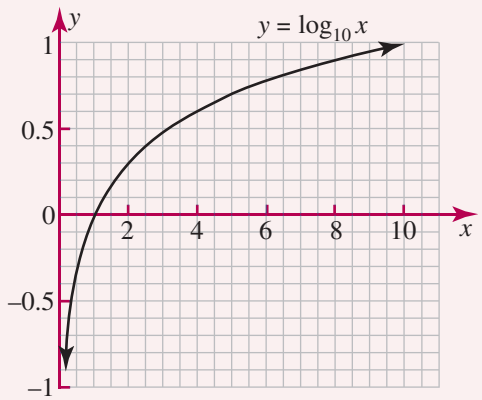
1 a $x = 2.81$, **b** $x = 1.54$, **c** $x = 4.32$ **2 a** $x = 2.322$, **b** $x = 1.262$, **c** $x = 1.365$, **d** $x = 0.565$, **e** $x = 0.301$, **f** $x = 2.073$, **g** $x = 3.287$, **h** $x = 5.644$, **i** $x = 3.145$, **j** $x = 3.336$, **k** $x = 4.247$, **l** $x = 9.054$ **3 a** $x = 1.335$, **b** $x = 4.585$, **c** $x = -1.663$, **d** $x = 1.269$, **e** $x = 1.465$, **f** $x = 0.171$, **g** $x = 2.953$, **h** $x = 0.193$, **i** $x = -1.202$
4 a $x = 1.756$, **b** $x = 2.593$, **c** $x = 12.457$, **d** $x = 7.129$, **e** $x = 2.821$, **f** $x = 2.379$ **5** $x < -1.585$

Exercise 12.9

1 a

x	0.2	0.4	0.6	0.8	1	2	3	4	5	6	7	8	9	10
y	−0.70	−0.40	−0.22	−0.10	0	0.30	0.48	0.60	0.70	0.78	0.85	0.90	0.95	1

b



c $x = 1$,
d i $x > 1$, **ii** $0 < x < 1$,
e no,
f i the y-values increase,
ii the y-values decrease,
g i 0.4, **ii** 0.6, **iii** 0.9,
h i 100, **ii** 1000, **iii** 10 000

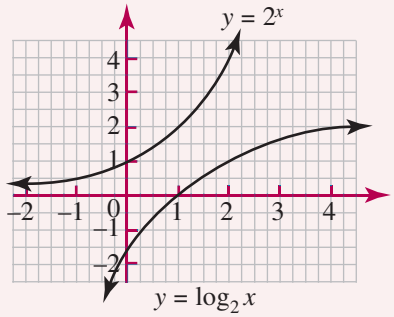
2 a

x	−3	−2	−1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

b definition of a logarithm, **c**

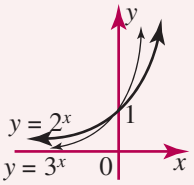
x	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
y	−3	−2	−1	0	1	2	3

d

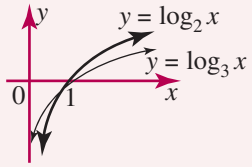


e The graphs are reflections of each other in the line $y = x$.
(That is the functions are inverses of each other.)

3 a



b



c **i** $\log_2 x$, **ii** $\log_3 x$ **4 a** $P(1, 0)$, $Q(2, 1)$, $R(4, 2)$, $S(8, 3)$

b $m_{PQ} = 1$, $m_{QR} = \frac{1}{2}$, $m_{RS} = \frac{1}{4}$, **c** the gradient decreases

5 a $A(9, 2)$, $B(0, 2)$, $C(1, 0)$, **b** 10.7 units^2

6 a $A(0, 1)$, $B(2, 1)$, $AB = 2$ units, **b** $C(16, 4)$, $D(2, 4)$, $CD = 14$ units

c 24 units^2

Chapter 12 Review

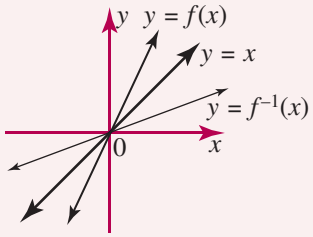
1 a yes, **b** no **2 a** no, **b** yes, **c** yes, **d** no, **e** yes, **f** no **3 a** $-1 \leqslant x \leqslant 1$; $-1 \leqslant y \leqslant 1$,

b all real x, $x \neq 2$; all real y, $y \neq 0$, **c** all real x; $y \geqslant -4$, **d** all real x; $y = 1, 3$, **e** all real x; $y \leqslant -2$ or $y > 2$,

f $x \geqslant 0$; all real y **4 a i** 4, **ii** 40, **iii** $2 - 3\sqrt{2}$, **iv** $-2\frac{1}{4}$, **b i** 32, **ii** 1, **iii** 8, **iv** $\sqrt{2}$ **5 a** $x = -3, 4$, **b** $x < -2$,

c $x = 64$ **6** $x + 5$ **7 a** no, **b** no, **c** yes, **d** yes **8 a** $y = \frac{1}{3}x$, **b** $y = x + 7$, **c** $y = 5x + 10$, **d** $y = 1 + \frac{3}{x}$

9 a



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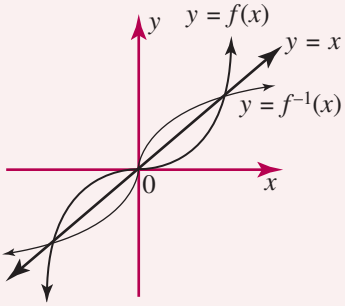
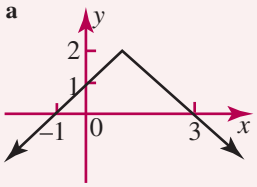


10 a



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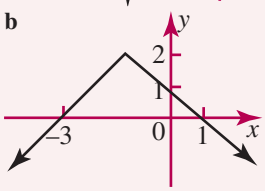
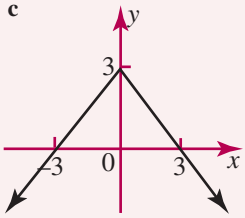
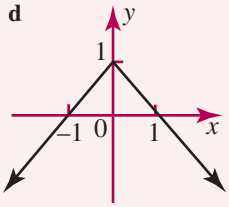


c



d



11 a $x = 5$, **b** $x = \frac{1}{2}$, **c** $x = -1$, **d** $x = -3$, **e** $x = \frac{3}{2}$, **f** $x = \frac{7}{2}$, **g** $x = \frac{3}{2}$, **h** $x = -1$ **12 a** $\log_3 81 = 4$, **b** $10^2 = 100$,

13 a $x = 3$, **b** $x = \frac{3}{4}$, **c** $x = -2$, **d** $x = \frac{1}{4}$ **14 a** $x = 7$, **b** $x = 3$, **c** $x = 6$, **d** $x = 8$ **15 a** 2, **b** 0, **c** -2, **d** $\frac{5}{2}$

16 a 2, **b** 1, **c** 3, **d** 1, **e** 3 **17 a** 1.079, **b** -0.125, **c** 1.204, **d** 1.556, **e** -0.477, **f** 0.301, **g** 1.602, **h** 2.477

18 a 11, **b** 3, **c** 0 **19 a** $x = 75$, **b** $x = 8$, **c** $x = 18$, **d** $x = 4$ **20 a** $k = 3$, **b** $A(0, 3)$, $B(8, 0)$, $C(1, 0)$, **c** $\sqrt{10}$ units

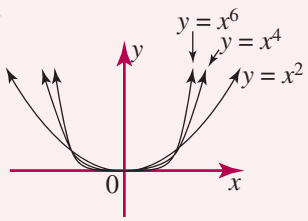
21 a $x = 1.92$, **b** $x = 4.46$, **c** $x = 1.12$

ANSWERS

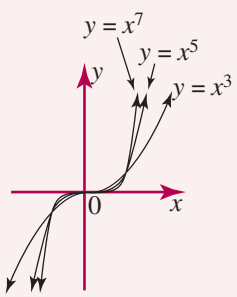
13 Curve sketching and polynomials

Exercise 13.1

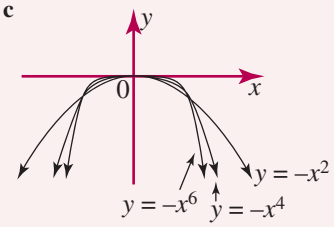
1 a



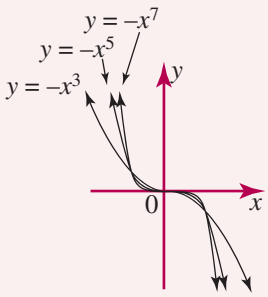
b



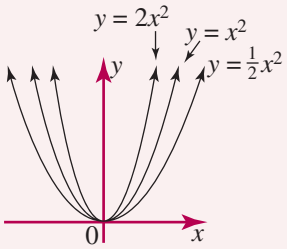
c



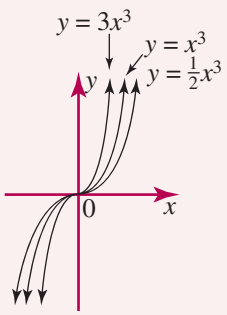
d



2 a



b



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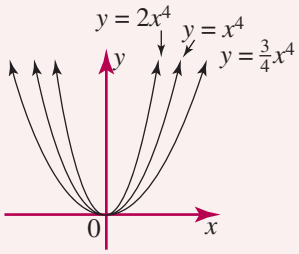
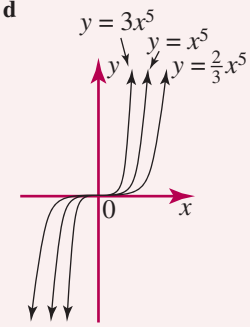
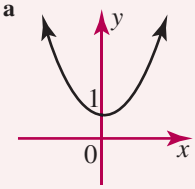


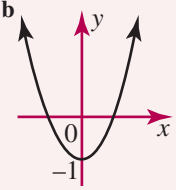
d



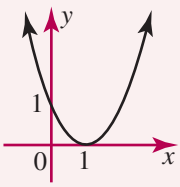
3 a



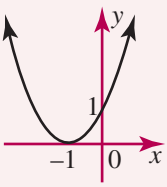
b



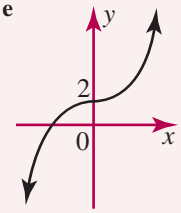
c



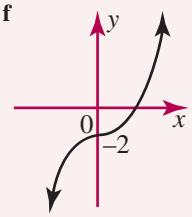
d



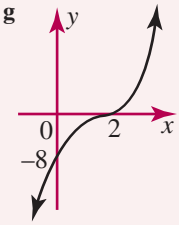
e



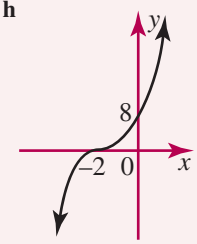
f



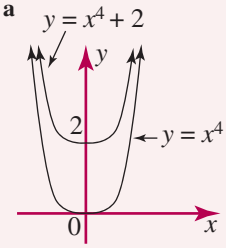
g



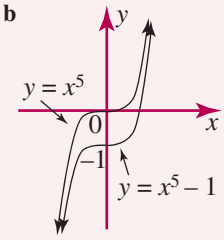
h



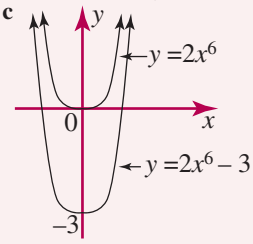
4 a



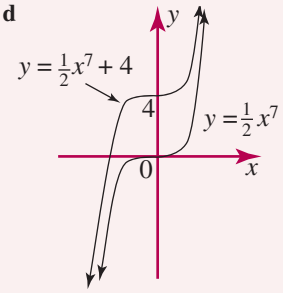
b



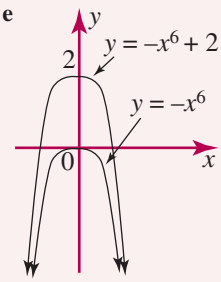
c



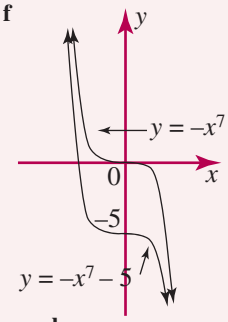
d



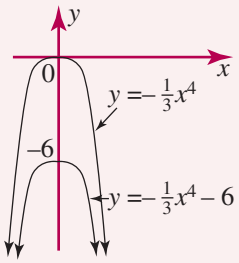
e



f



g



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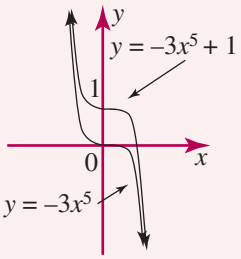
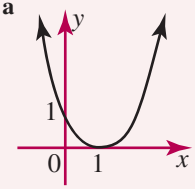
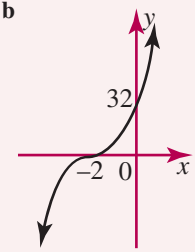


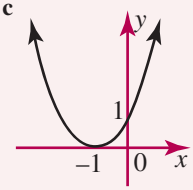
5 a



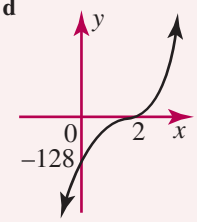
b



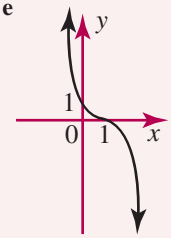
c



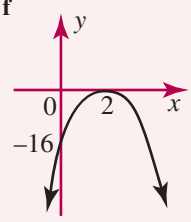
d



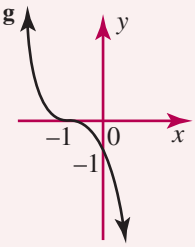
e



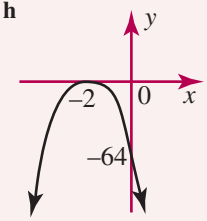
f



g

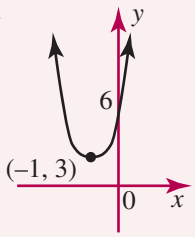


h

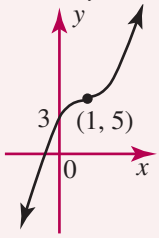


6 **a** $x = -2$, $y = 32$, **b** $x = 2$, $y = -96$, **c** $x = 1$, $y = -3$, **d** $x = -1$, $y = -2$

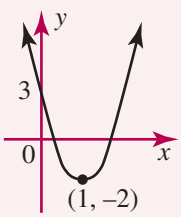
7 a



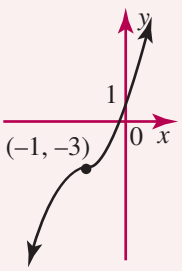
b



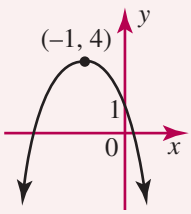
c



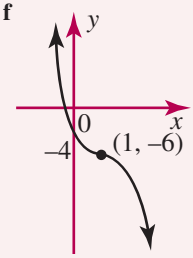
d



e



f



Exercise 13.2

1 **a** $x^2 + y^2 = 9$, **b** $x^2 + y^2 = 49$, **c** $x^2 + y^2 = 2$, **d** $x^2 + y^2 = 20$ **2** **a** 5, **b** 8, **c** 12, **d** $\sqrt{3}$, **e** $3\sqrt{2}$, **f** $4\sqrt{2}$

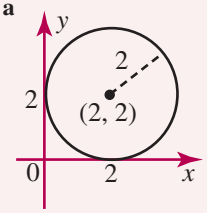
3 $x^2 + y^2 = \frac{49}{4}$, centre is (0, 0), radius is $3\frac{1}{2}$ units. **4** **a** (1, 5), $r = 3$, **b** (–6, –2), $r = 2$, **c** (–2, 3), $r = 5$,

d (4, –5), $r = 1$, **e** (0, 7), $r = 6$, **f** (–3, 0), $r = 10$, **g** (–4, –6), $r = \sqrt{5}$, **h** (2, –1), $r = 2\sqrt{2}$, **i** (0, 4), $r = 2\sqrt{10}$,

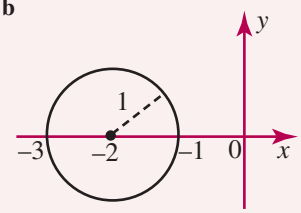
ANSWERS

j $(10, 0)$, $r = 3\sqrt{5}$ **5 a** $(x-1)^2 + (y-3)^2 = 25$, **b** $(x+2)^2 + (y-7)^2 = 36$, **c** $(x+4)^2 + (y+1)^2 = 9$, **d** $(x-6)^2 + (y+3)^2 = 81$, **e** $(x-2)^2 + (y-1)^2 = 7$, **f** $(x+3)^2 + (y+2)^2 = 15$, **g** $(x-7)^2 + (y+5)^2 = 18$, **h** $(x+8)^2 + (y-11)^2 = 28$ **6 a** $x^2 + y^2 = 49$, **b** $x^2 + y^2 = 2$, **c** $(x-4)^2 + (y-6)^2 = 9$, **d** $(x+9)^2 + (y-12)^2 = 4$, **e** $(x-3)^2 + y^2 = 9$, **f** $x^2 + (y+6)^2 = 25$

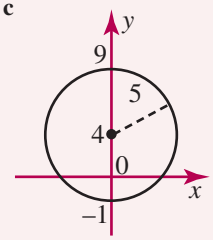
7 a



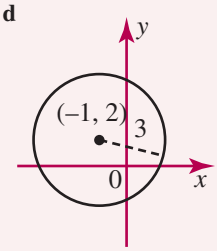
b



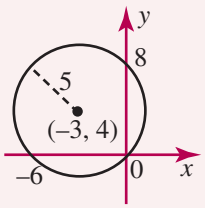
c



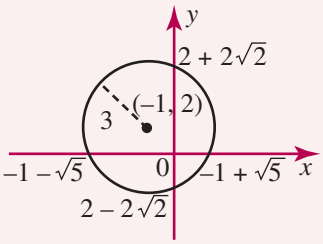
d



8 a



b



9 a yes, **b** no, **c** no, **d** no, **e** yes, **f** yes, **g** no, **h** no, **i** no, **j** yes, **k** no, **l** yes

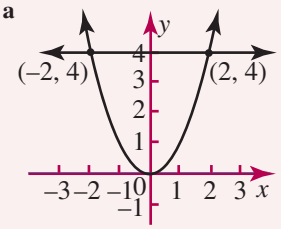
10 a $(1, 2)$, $r = 5$, **b** $(-6, -5)$, $r = 3$, **c** $(9, -7)$, $r = 8$, **d** $(-2, 9)$, $r = 11$, **e** $(0, -3)$, $r = 7$, **f** $(7, 0)$, $r = 8$, **g** $(-3, 5)$, $r = 3\sqrt{3}$, **h** $(10, -2)$, $r = 2\sqrt{11}$

11 a $y = \sqrt{1-x^2}$, **b** $y = -\sqrt{1-x^2}$, **c** $y = \sqrt{16-x^2}$, **d** $y = \sqrt{49-x^2}$, **e** $y = -\sqrt{3-x^2}$, **f** $y = -\sqrt{24-x^2}$

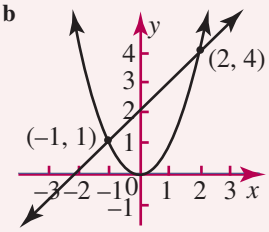
12 a 3, **b** 2, **c** 5, **d** 6, **e** 10, **f** $\sqrt{6}$, **g** $2\sqrt{10}$, **h** $3\sqrt{7}$

Exercise 13.3

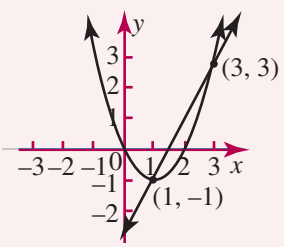
1 a



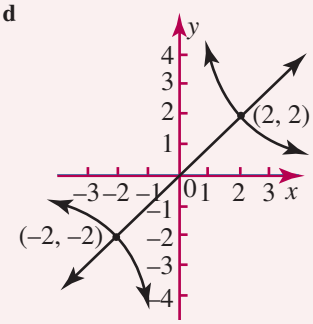
b



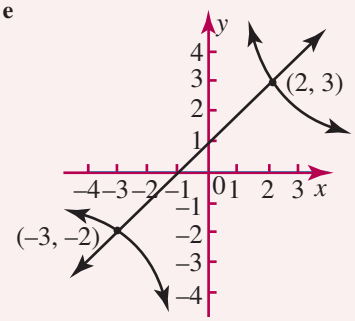
c



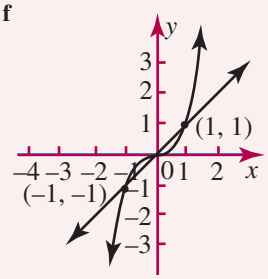
d



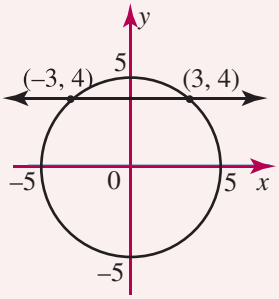
e



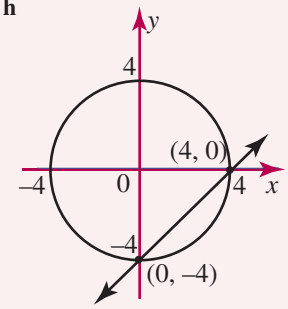
f



g

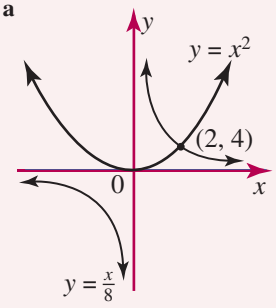


h

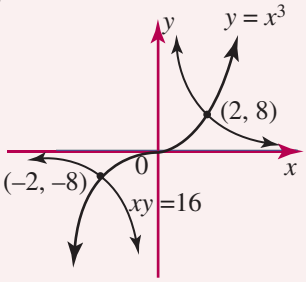


2 **a** (5, 25), (–5, 25), **b** (0, 0), (3, 9), **c** (–3, 3), (1, 3), **d** (3, 6), (5, 20)
3 **a** (3, 2), **b** (–2, –4), (2, 4), **c** (–6, –2), (6, 2), **d** (1, 3), (3, 1)
4 **a** (–5, 3), (5, 3), **b** (–5, –5), (5, 5), **c** (–2, 4), (2, –4), **d** (–2, –6), (6, 2)

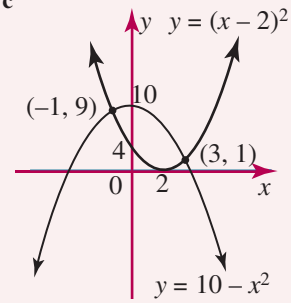
5 **a**



b



c



6 (–1, –1), (1, 1) **7** **a** (–2, 4), (2, 4), **b** (1, 2), (–1, –2), (2, 1), (–2, –1)

Exercise 13.4

1 **a** yes, **b** yes, **c** no, **d** no, **e** yes, **f** no, **g** yes, **h** yes, **i** no, **j** no, **k** yes, **l** no, **m** yes, **n** no, **o** no, **p** yes, **q** no, **r** no, **s** yes, **t** yes, **u** no **2** B, C, E, H, I **3** **a** **i** 3, **ii** x^3, **iii** 1, **iv** 4, **b** **i** 4, **ii** $2x^4$, **iii** 2, **iv** –1, **c** **i** 6, **ii** $5x^6$, **iii** 5, **iv** 12, **d** **i** 5, **ii** $3x^5$, **iii** 3, **iv** 0, **e** **i** 1, **ii** $-4x$, **iii** –4, **iv** 7, **f** **i** 4, **ii** x^4, **iii** 1, **iv** 11, **g** **i** 2, **ii** $-3x^2$, **iii** –3, **iv** 0, **h** **i** 0, **ii** 4, **iii** 4, **iv** 4, **i** **i** 3, **ii** $\sqrt{2}x^3$, **iii** $\sqrt{2}$, **iv** –10, **j** **i** 4, **ii** $\frac{1}{2}x^4$, **iii** $\frac{1}{2}$, **iv** –8, **k** **i** 3, **ii** $\frac{x^3}{2}$, **iii** $\frac{1}{2}$, **iv** 0, **l** **i** 6, **ii** $\frac{5x^6}{3}$, **iii** $\frac{5}{3}$, **iv** 1 **4** **a** **i** –3, **ii** 11, **iii** –9, **b** **i** 30, **ii** 18, **iii** 2, **c** **i** 265, **ii** –63, **iii** $1\frac{3}{4}$, **d** **i** 10, **ii** 13, **iii** $4\frac{2}{3}$ **5** **a** $x = -3, 3$, **b** $x = -1, 3$, **c** $x = -5, 3$, **d** $x = 0, 1, -1$, **e** $x = 0, 2, -2$, **6** **a** **i** 2, **ii** $9x^2$, **iii** 9, **iv** 16, **b** **i** 3, **ii** $5x^3$, **iii** 5, **iv** 0, **c** **i** 4, **ii** $4x^4$, **iii** 4, **iv** 0, **d** **i** 3, **ii** x^3, **iii** 1, **iv** –8, **e** **i** 12, **ii** $16x^{12}$, **iii** 16, **iv** 1, **f** **i** 3, **ii** x^3, **iii** 1, **iv** 24, **g** **i** 5, **ii** $2x^5$, **iii** 2, **iv** 10, **h** **i** 2, **ii** $2x^2$, **iii** 2, **iv** 37, **i** **i** 1, **ii** $20x$, **iii** 20, **iv** 20, **j** **i** 4, **ii** $13x^4$, **iii** 13, **iv** 29 **7** **a** $k = 7$, **b** $k = 3, 4$ or 5, **c** $k \geqslant 6$ **8** **a** $p \neq 1$, q and r = any value, **b** $p = 1$, $q \neq -4$, r = any value, **c** $p = 1$, $q = -4$, $r \neq 5$, **d** $p = 1$, $q = -4$, $r = 5$

Exercise 13.5

1 **a** $x^3 + 7x^2 - x + 16$, **b** $x^3 + 2x^2 + 1$, **c** $2x^3 + 5x^2 + 5x + 3$, **d** $x^3 + 3x^2 + 5x + 2$, **e** $x^3 - 2x^2 + 6x - 13$, **f** $5x^2 - x + 15$, **g** $2x^3 + 7x^2 + 2x + 10$, **h** $-3x^2 - 2x - 8$, **i** $4 - 8x - 3x^2 - 2x^3$ **2** **a** $x^3 + 6x^2 + 3x + 7$, **b** $4x^3 - 5x^2 + 4x - 10$, **c** $7x^3 - 10$, **d** $x^3 + 6x^2 + 15x - 13$, **e** $x^3 - 13x^2 + 7x - 9$, **f** $x^4 - 5x^3 - 7x^2 + 16x + 14$, **g** $x^4 - 4x^3 + 8x^2 - 18$, **h** $2x^3 - 6x^2 + 10x - 2$, **i** $2x^3 + 4x^2 - 2x + 1$ **3** **a** $x^3 + 5x^2 + 6x + 2$, **b** $x^3 - 5x^2 + 2x + 8$, **c** $9 - 24x + 10x^2 - x^3$, **d** $3x^3 + 7x^2 - 2x - 8$, **e** $6x^3 + 7x^2 - 24x - 10$, **f** $8x^4 + 26x^3 - 24x^2 - 4x + 3$ **4** **a** 5, **b** 5, **c** 8, **d** 10, **e** 5, **f** 7 **5** **a** 4, **b** 0 **6** **a** m, **b** m, **c** $m + n$ **7** **a** $2x^5 - 9x^3 + 8x^2 - 35x + 20$ **8** **a** $x^4 - x^3 + 2x$, **b** **i** $2x^4$, **ii** $2x^3 - 4x$, **iii** $x^8 - x^6 + 4x^4 - 4x^2$ **9** $x^3 - 2x^2 - x - 28$, **10** $m = 5$, $n = -6$

Exercise 13.6

1 **a** $x + 4$, **b** $x - 3$, **c** $2x - 7$, **d** $5x + 2$ **2** **a** $(x + 2)(x + 5) + 4$, **b** $(x - 4)(x + 8) + 3$, **c** $(3x + 2)(2x + 5) + 7$, **d** $(2x - 5)(5x - 2) - 4$ **3** **a** $(x + 1)(x^2 + 6x + 4) + 11$, **b** $(x + 3)(x^2 + 9x + 10) + 2$, **c** $(x - 2)(x^2 - 7x + 2) + 1$, **d** $(x - 5)(x^2 + x - 4) + 14$, **e** $(x + 4)(2x^2 + 7x + 4) - 1$, **f** $(3x + 5)(x^2 - 4x + 3) - 7$, **g** $(3x - 4)(x^2 + 4) + 12$, **h** $(2x + 7)(3x^2 - x - 3) + 11$, **i** $(x + 2)(x^3 - 7x^2 + 17x - 22) + 14$, **j** $(2x - 3)(x^3 + 2x^2 - 5x - 2) - 8$
4 **a** $Q(x) = x^2 + 4x - 4$, $R(x) = 12$, **b** $Q(x) = x^2 + 2x + 1$, $R(x) = 6$, **c** $Q(x) = x^3 + x^2 - 2x - 2$, $R(x) = 5$, **d** $Q(x) = x^3 - 3x^2 + 9x - 23$, $R(x) = 63$ **5** **a** $Q(x) = x^2 + 2x - 8$, $R(x) = 4x + 11$,

ANSWERS

b $Q(x) = x^2 + 8x + 21$, $R(x) = 48x - 17$, **c** $Q(x) = 2x^2 + 9x + 1$, $R(x) = -7x - 5$. **d** $Q(x) = x^2 + 5x - 3$, $R(x) = -14x + 7$, **e** $Q(x) = 4x^2 + 3x + 10$, $R(x) = 8x + 62$, **f** $Q(x) = x^2 - 2x + 7$, $R(x) = -8x - 16$, **g** $Q(x) = 2x^2 - 8x + 9$, $R(x) = -12x - 30$, **h** $Q(x) = 6x^2 + 3x + 4$, $R(x) = 4x - 4$ **6 a** $Q(x) = x^2 + 6x + (k + 6)$, $R(x) = 3k + 6$, **b** $k = -2$, **c** $k = 5$

Exercise 13.7

1 a 3 **2 a** 11, **b** −6, **c** 4, **d** 10, **e** 7, **f** 5, **g** 14, **h** −15 **3** $k = -7$ **4 a** $k = -7$, **b** $k = 7$, **c** $k = -5$

5 $a = 1$, $P(x) = x^3 + x^2 + x - 6$ **6 a** $5\frac{1}{8}$, **b** $7\frac{1}{27}$ **7** $k = -5, 3$ **8** $P(x) = 2(x + 1)(x - 2)(x - 4)$

9 a $P(x) = -(x + 5)(x + 2)(x - 1)(x - 3)$, **b** $P(x) = -2(2x + 1)(3x - 2)(x - 2)$ **10 a** the divisor is of degree 2, so any remainder must be of degree 1 or 0. **b** $a = 3$, $b = 5$, $R(x) = 3x + 5$ **11** $R(x) = 2x - 4$

Exercise 13.8

1 a $P(2) = 0$ **2 a** yes, **b** no, **c** no, **d** yes, **e** no, **f** yes **3 a** $(x - 1)(x - 2)(x + 4)$, **b** $(x + 1)(x - 7)(x + 3)$, **c** $(x + 6)(x - 1)(x + 1)$, **d** $(x - 2)(x - 5)(x + 3)$, **e** $(x + 1)(x - 4)(x + 2)$, **f** $(x + 3)(x - 2)(x - 1)$

4 a $(x - 2)(3x - 4)(x + 3)$, **b** $(x + 2)(2x - 1)(x - 6)$ **5 a** $(x - 1)(x + 4)(x + 5)$, **b** $(x + 1)(x - 2)(x + 7)$, **c** $(x - 2)(x + 2)(x - 3)$, **d** $(x + 2)(x + 3)(x + 5)$, **e** $(2x + 5)(x - 1)(x + 2)$, **f** $(3x - 2)(x + 3)(x - 4)$

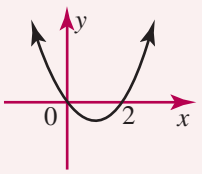
6 $(x - 2)^2(x + 1)$ **7 a** $x = 1, 8, -3$, **b** $x = 2, -2, -4$, **c** $x = 3, 5, -3$, **d** $x = 2, 3, 7$, **e** $x = -1, 4, \frac{1}{2}$, **f** $x = 2, 5, -\frac{2}{3}$

8 a $x = -1, 1 \pm \sqrt{3}$, **b** $x = 2, 1 \pm \sqrt{5}$ **9** $x = -1$ **10 a** $k = 30$, **b** $k = -11$, **c** $k = -3$, **d** $k = 12$ **11** $x = -1, 2, 5$

12 $a = -1, -4, 7$ **13** $a = -6$, $b = 30$ **14** $p = 4$, $q = -15$

Exercise 13.9

1 a



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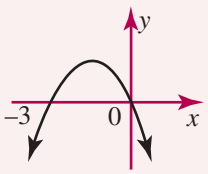

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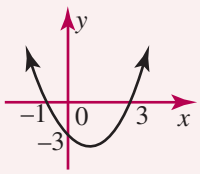

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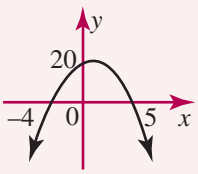

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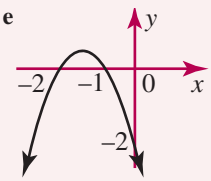

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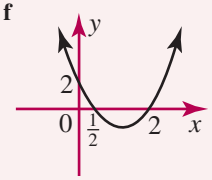

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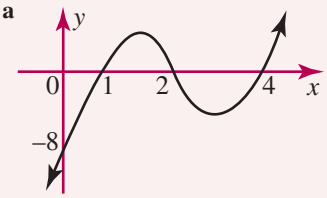

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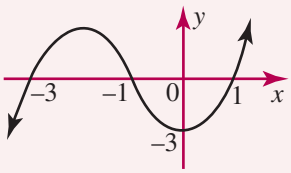

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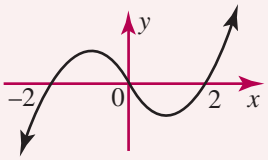

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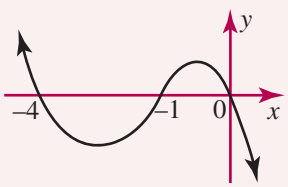

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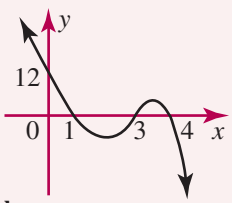

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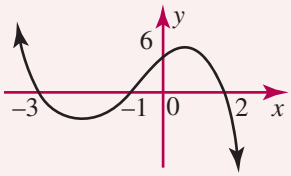

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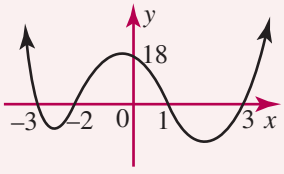

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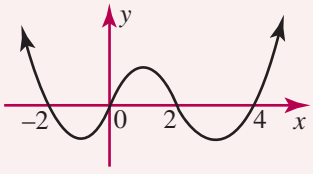

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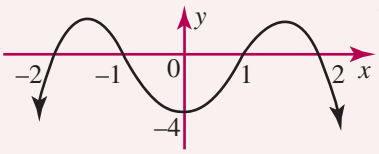

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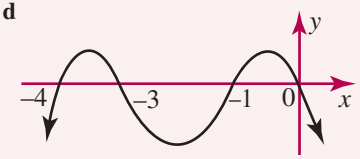

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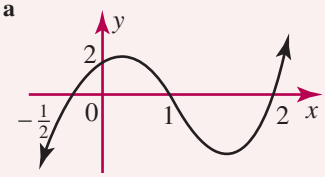

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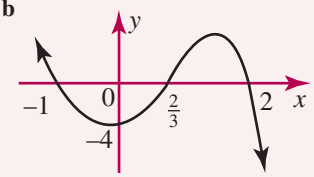

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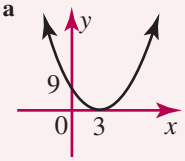

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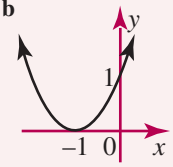

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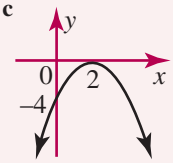

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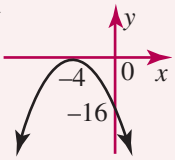

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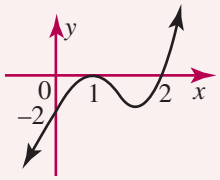

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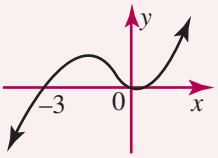

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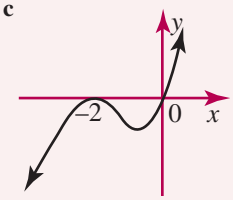

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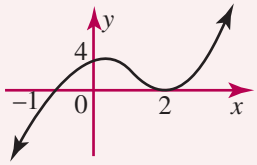

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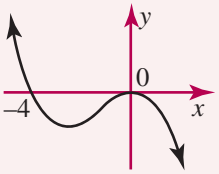

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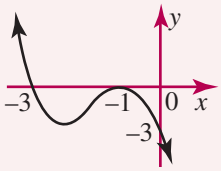

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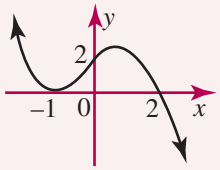

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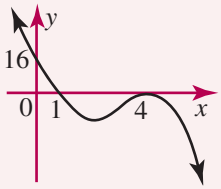

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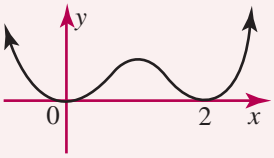

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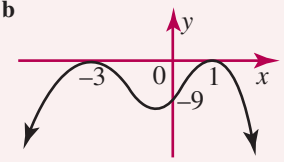

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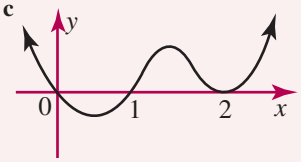

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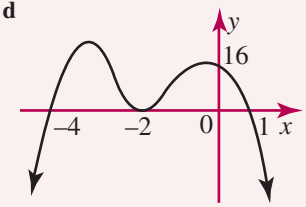

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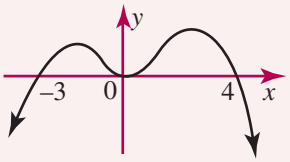

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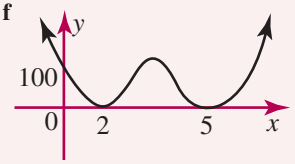

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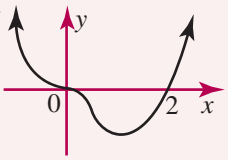

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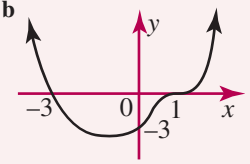

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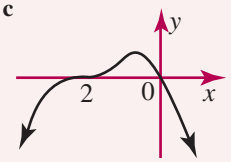

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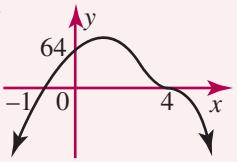

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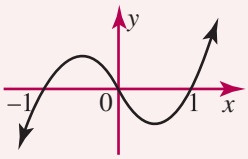

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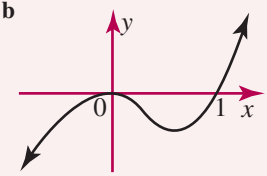

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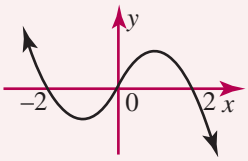

ANSWERS

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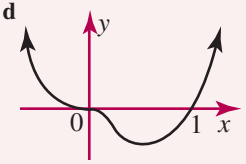

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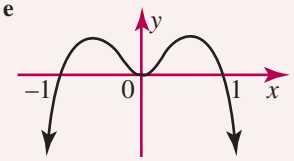

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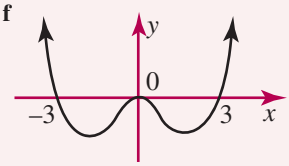

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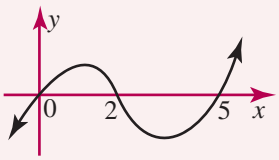

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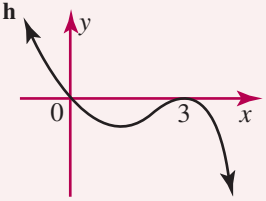

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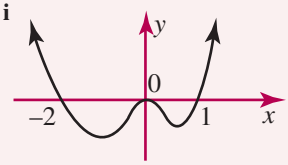

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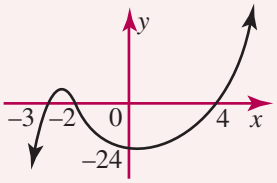

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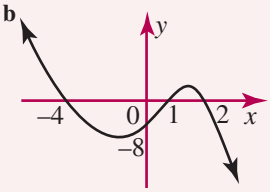
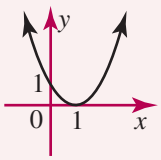


11 **b** once

12 **a** $y = 2(x + 2)(x - 2)(x - 3)$,
b $y = -4x(x + 3)(x - 3)$,
c $y = 3(x + 4)(x + 1)(x - 2)$,
d $y = (x + 1)(x - 3)^2$,
e $y = -2(x + 3)^2(x - 2)$,
f $y = 4(x + 2)^2(x - 1)$

Exercise 13.10

1 a



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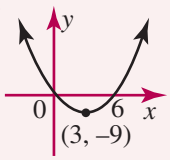

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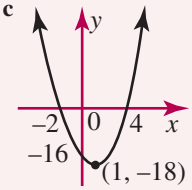

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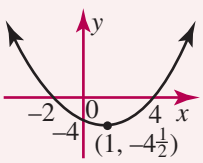

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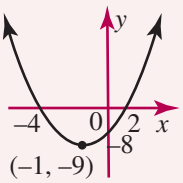

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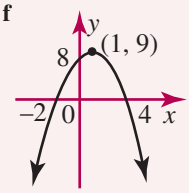

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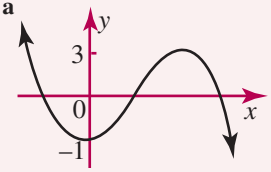

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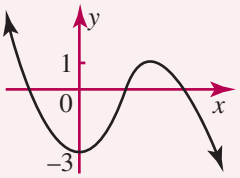

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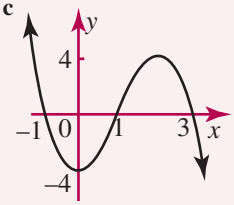

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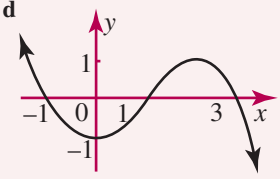

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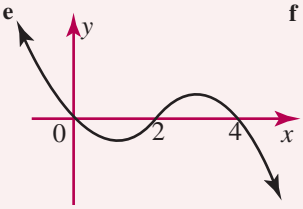

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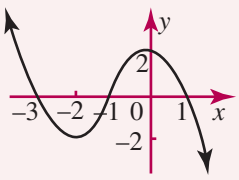

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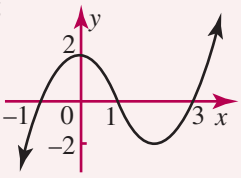

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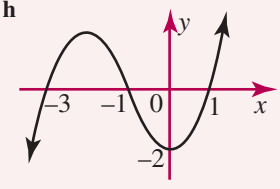

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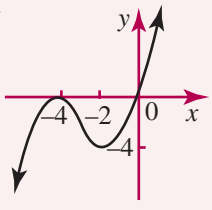

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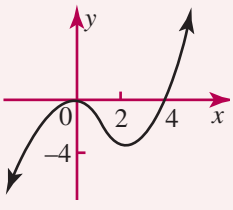

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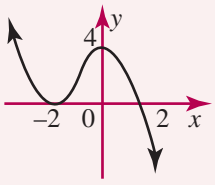

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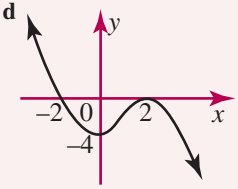

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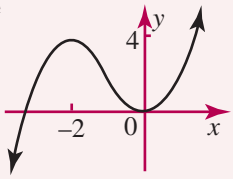

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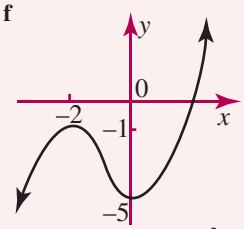

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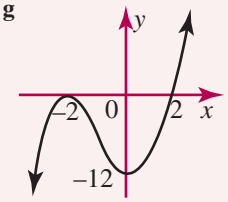

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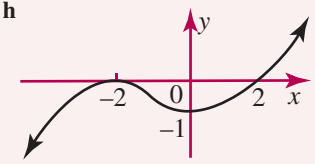

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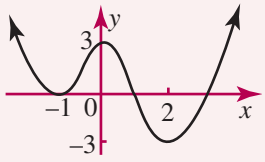

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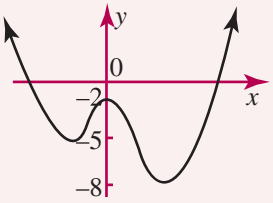

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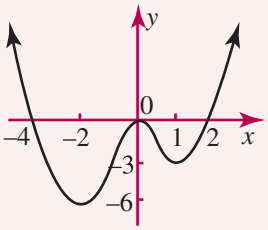

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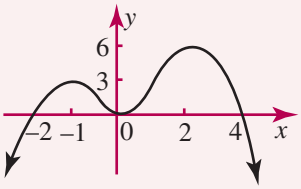

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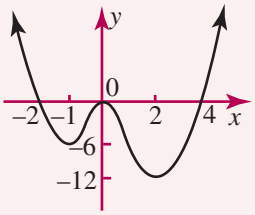

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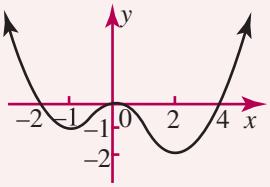

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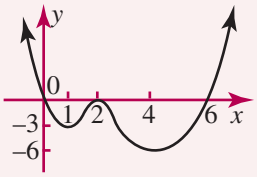

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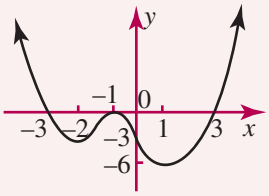

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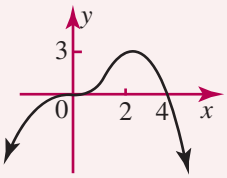

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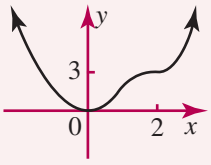

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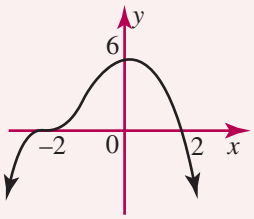

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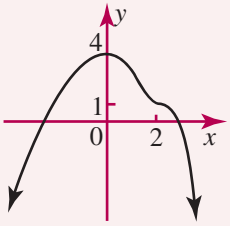
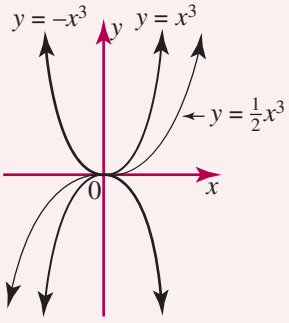


Chapter 13 Review

1 a



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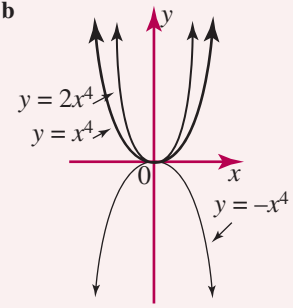

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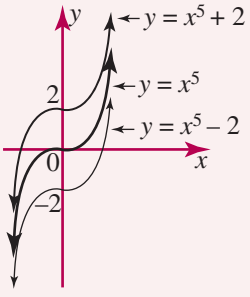
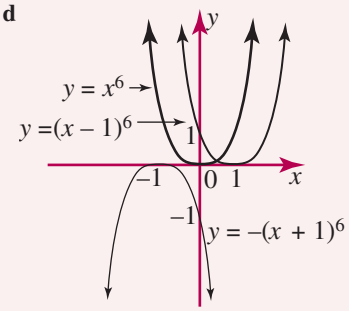


ANSWERS

d



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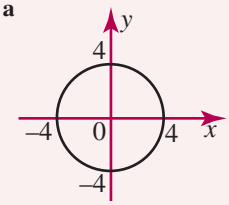

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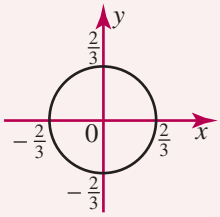


3 **a** $(2, -3)$, $r = 7$, **b** $(-6, -1)$, $r = 2$, **c** $(0, 4)$, $r = 2\sqrt{2}$ **4** **a** $(x-3)^2 + (y+5)^2 = 64$, **b** $(x+2)^2 + y^2 = 12$

5 **a** yes, **b** no, **c** yes, **d** no, **e** yes **6** **a** $(1, -3)$, $r = 4$, **b** $(5, 0)$, $r = 3\sqrt{5}$ **7** **a** $(1\frac{1}{2}, -7\frac{1}{2})$, $(5, 10)$,

b $(-4, -3)$, $(3, 4)$, **c** $(3, 6)$, $(-3, -6)$ **8** **a** yes, **b** no, **c** no, **d** yes, **e** no, **f** yes, **g** yes, **h** yes **9** **a** **i** $5x^4$, **ii** 5,

iii 4, **iv** 7, **b** **i** $-x^5$, **ii** -1, **iii** 5, **iv** 6, **c** **i** x^2, **ii** 1, **iii** 2, **iv** 0, **c** **i** $\frac{x^3}{5}$, **ii** $\frac{1}{5}$, **iii** 3, **iv** -1

10 the leading co-efficient is 1 in $x^3 - 4x$, but not in $4x - x^3$. **11** **a** 4, **b** -8, **c** 4, **d** $6\frac{2}{27}$, **e** $8\sqrt{2} - 6$,

f $27a^3 - 45a^2 + 18a + 4$ **12** **a** $2x^3 + 3x^2 - 10x + 10$, **b** $7x^2 + 2x + 4$ **13** $6x^4 + 8x^3 - 9x^2 + 3x + 20$

14 **a** $4x^3 + 7x^2 + 7x - 11$, **b** $2x + 1$, **c** $14x^2 + 2x - 23$, **d** $1 - 3x - 4x^2$, **e** $28x^5 + 8x^4 - 9x^3 + 10x^2 - 55x$,

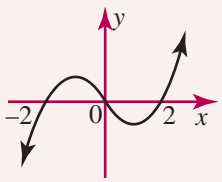
f $8x^4 + 48x^3 + 10x^2 + 60x$ **15** **a** $(x-1)(x-3) + 2$, **b** $(x+2)(x^2 + 3x - 7) + 20$, **c** $(2x-3)(x^2 + 3x + 1) - 7$,

d $(x-2)(x^3 + 2x^2 + 10x + 20) + 50$ **16** **a** -2, **b** 12, **c** 3, **d** 18 **17** **a** $k = 4$, **b** $k = -4$ **18** **a** $P(4) = 0$,

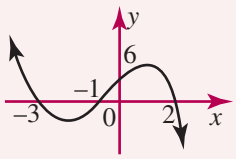
b $P(-2) = 0$ **19** $k = -3$ **20** **a** $P(-2) = 0$, **b** $(x+2)(x+4)(x-3)$ **21** **a** $(x-2)(x-3)(x+7)$,

b $(2x+1)(x-1)(x-3)$ **22** $x = \frac{2}{3}, -1, -2$ **23** $a = -10, b = 31$

24 **a**



b



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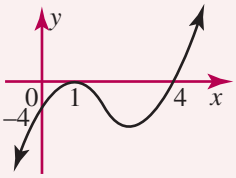

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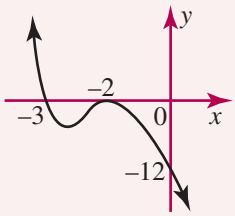
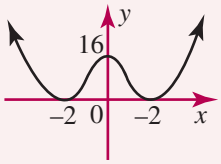
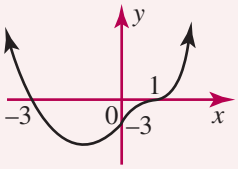


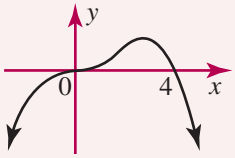
e



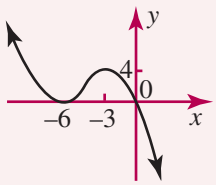
f



g



25 **a**



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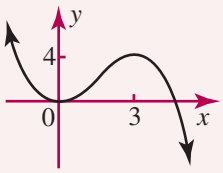

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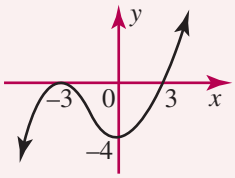
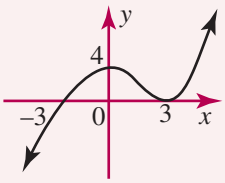


d



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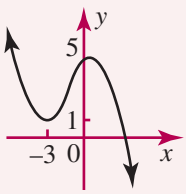

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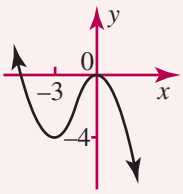
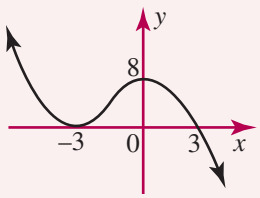
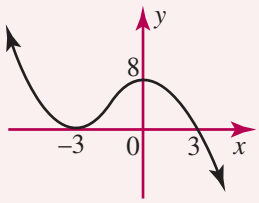


g



h



14 Circle geometry

Exercise 14.1

1 **a** radius, **b** diameter, **c** circumference, **d** chord, **e** minor arc, **f** major arc, **g** minor segment, **h** major segment, **i** minor sector, **j** major sector, **k** tangent, **l** secant **2** **a** radius, **b** diameter, **c** tangent, **d** chord, **e** semi-circle, **f** secant, **g** sector, **h** segment **3** **a** OA, OB, OE, **b** AB, CD, CE, DE **4** **a** semi-circle, **b** arc, **c** chord, **d** radius, **e** tangent, **f** diameter, **g** segment, **h** secant, **i** sector **5** **a** 24 cm, **b** diameter **6** **a** equal radii, **b** isosceles, **c** $\angle OJK = \angle OKJ$ **7** equilateral, all angles are 60° **8** 4 cm **9** 3 cm **10** **a** $\angle POQ$, **b** $\angle ROS$ **11** **a** **i** $\angle JNK$, $\angle JMK$, $\angle JLK$, **ii** $\angle LNM$, $\angle LJM$, $\angle LKM$, **b** **i** $\angle JKM$, $\angle JLM$, $\angle JNM$ **ii** $\angle LJN$, $\angle LKN$, $\angle LMN$ **12** **a** $\angle TUW$, $\angle TVW$, **b** **i** $\angle TSW$, **ii** $\angle TUV$, **iii** $\angle STU$ **13** **a** GE, GN, **b** XC, XR **14** $\angle AEC$, $\angle ADC$ **15** no, PR is not a diameter **16** **a** 30°, **b** 60°, **c** 60°, **d** 90° **17** **a** $\angle POR$, **b** $\angle PTR$, $\angle PQR$, $\angle PSR$ **18** **a** yes, **b** cyclic quadrilateral, **c** $\angle M$ **19** no, the points O, X, Y, Z are not concyclic **20** $\angle RQT$ **21** **a** rhombus, **b** kite **22** yes **23** construct the perpendicular bisectors of the chords and find their point of intersection

Exercise 14.2

1 **a** 7 cm, **b** 10 cm, **c** 9 cm, **d** 6.5 cm, **e** 25 cm, **f** 8 cm **2** **a** $x = 52$, **b** $p = 11$, **c** $c = 60$ **3** **a** 45°, **b** $VW = 4$ cm, $XY = 7$ cm, $UZ = 6$ cm, **c** 108° **4** **a** 5 cm, **b** 14 cm, **c** $WX = 22$ cm, $WM = 11$ cm, **d** $GH = 15$ cm, $OQ = 9$ cm, **e** 6.5 cm, **f** 6 cm **5** **a** $MD = 12$ cm, $CD = 24$ cm, **b** 29 cm, **c** 48 cm, **d** $OM = 9$ cm, $HM = 24$ cm, **e** $OZ = 25$ cm, $WX = 40$ cm, **f** $OM = 30$ cm, $MN = 16$ cm, **g** 9 cm, **h** $QR = 16$ cm, $PR = 56$ cm, **i** 35 cm

6 **a** 34 cm, **b** 48 cm **7** $10\sqrt{2}$ cm **8** **a** 7 cm, **b** 23 cm **9** 21 cm **10** 48 cm

Exercise 14.3

1 **a** $a = 120$, **b** $e = 52$, **c** $y = 45$, **d** $w = 24$, **e** $q = 56$, **f** $b = 146$, **g** $d = 43$, **h** $n = 37$, **i** $x = 33$ **2** **a** $m = 40$, **b** $z = 25$, **c** $c = 118$, **d** $p = 41$, $q = 41$, **e** $x = 70$, $y = 70$, **f** $e = 110$, $f = 55$, **g** $u = 42$, $v = 84$, **h** $j = 37$, $k = 37$, **i** $r = 28$, $s = 28$, $t = 56$ **3** **a** $n = 90$, **b** $a = 180$, **c** $u = 90$, $v = 90$ **4** **a** a = 50, $b = 45$, **b** $m = 90$, $n = 65$, **c** $t = 31$, $u = 84$, **d** $g = 67$, $h = 134$, **e** $x = 46$, $y = 46$, $z = 92$, **f** $e = 150$, $f = 75$, **g** $r = 28$, s = 56, **h** $c = 65$, $d = 65$, **i** $v = 70$, $w = 140$, **j** $j = 42$, $k = 42$, **k** a = 220, $b = 110$, **l** $x = 62$, $y = 57$, $z = 31$, $w = 30$, **m** $a = 31$, $b = 62$, **n** $r = 72$, $s = 54$, **o** $v = 53$, $w = 53$ **5** **a** $w = 63$, **b** $q = 38$, **c** $z = 57$, **d** $t = 158$, **e** $x = 59$, **f** $s = 113$ **6** $a = 25$, $b = 34$, $c = 42$

Exercise 14.4

1 **a** $x = 130$, **b** $k = 65$, **c** $m = 90$, **d** $p = 45$, $q = 128$, **e** $a = 112$, $b = 84$, **f** $u = 76$, $v = 53$, **g** $t = 90$, **h** $y = 60$, **i** $y = 35$ **2** **a** $w = 70$, **b** $e = 53$, **c** $g = 48$, **d** $k = 22$, **e** $u = 47$, **f** $r = 17$ **3** **a** no, the points O, A, B, C are not concyclic, **b** no, **c** 210°, **d** $x = 105$ **4** **a** $m = 65$, $n = 115$, **b** $e = 82$, $f = 98$, **c** $p = 121$, $q = 118$, **d** $c = 62$, $d = 62$, **e** $x = 72$, $y = 144$, **f** $r = 135$, $s = 45$ **5** **a** $g = 77$, $h = 154$, **b** $v = 74$, $w = 74$, **c** $a = 124$, $b = 56$, **d** $q = 51$, $r = 78$, $s = 102$, **e** $j = 64$, $k = 64$, **f** $s = 62$, $t = 118$, **g** $c = 27$, $d = 153$, **h** $e = 23$, $f = 23$, **i** $m = 90$, $n = 90$ **6** **a** $x = 55$, $y = 51$, $z = 51$, **b** $a = 41$, $b = 110$, $c = 110$, **c** $p = 65$, $q = 72$, $r = 108$, **d** $u = 90$, $v = 63$, $w = 117$, **e** $x = 43$, $y = 62$, $z = 56$, **f** $a = 125$, $b = 125$, **g** $g = 39$, $h = 78$, **h** $p = 64$, $q = 26$, $r = 50$, $s = 92$, **i** $a = 47$, $b = 34$, **j** $d = 58$, **k** $s = 51$, **l** $c = 17$, $d = 73$, $e = 107$, **m** $x = 23$, $y = 62$, $z = 118$, **n** $m = 56$, **o** $x = 46$, $y = 148$ **7** $p = 84$, $q = 67$ **8** 36° **9** $w = 114$, $x = 66$, $y = 48$, $z = 132$

Exercise 14.5

1 **a** $a = 90$, **b** $y = 38$, **c** $e = 56$ **2** **a** $x = 12$, **b** $w = 65$, **c** $g = 55$, **d** $q = 74$, **e** $t = 90$, $u = 90$, $v = 134$, **f** $e = 67$, $f = 15$ **3** **a** 11 cm, **b** 14 cm, **c** 44 cm, **d** 9 cm **4** **a** 37 cm, **b** 24 cm, **c** 24 cm, **d** 64 cm **5** **a** $x = 50$, **b** $c = 49$, $d = 52$, **c** $p = 68$, $q = 41$, **d** $g = 44$, $h = 82$, **e** $u = 108$, $v = 17$, **f** $e = 78$, $f = 38$, **g** $c = 33$, **h** $z = 156$, **i** $s = 60$ **6** **a** $a = 53$, $b = 106$, **b** $m = 66$, $n = 66$, **c** $e = 74$, $f = 74$, **d** $p = 35$, $q = 35$, **e** $x = 90$, $y = 78$, $z = 90$, $w = 78$, **f** $r = 81$, $s = 162$, $t = 9$, **g** $g = 43$, $h = 137$, **h** $a = 57$, $b = 47$, $c = 47$, $d = 76$, **i** $x = 77$, $y = 77$, $z = 26$ **7** **a** $b = 59$, **b** $f = 38$, **c** $c = 72$, **d** $t = 47$, **e** $m = 76$, **f** $k = 52$, **g** $r = 51$, **h** $x = 68$, **i** $y = 56$, **j** $v = 9$, **k** $j = 42$, **l** $e = 49$, **m** $s = 99$, **n** $a = 37$, **o** $d = 83$ **8** 37 cm **9** $x = 50$ **10** $x = 36$

Exercise 14.6

1 **a** $m = 10$, **b** $y = 16$, **c** $u = 6$ **2** **a** $p = 10$, **b** $j = 9$, **c** $c = 14$, **d** $u = 21$ **3** **a** $a = 4$, **b** $t = 6$, **c** $n = 12$, **d** $w = 7$ **4** **a** $x = 10$, $y = 5$, **b** $p = 12$, $q = 9.6$, **c** $m = 5$, $n = 30$, **d** $e = 12$, $f = 18$, **e** $r = 15$, $s = 40$ **5** **a** 24 cm, **b** 14 cm **6** $a = 9$, $b = 15$ **7** **a** $n = 4$, **b** $a = 1$, **c** $t = 4$, **d** $c = 5$ **8** 10 cm **9** **a** 6 cm, **b** 6 cm

ANSWERS

Exercise 14.7

1 **a** $x = 36$, **b** $e = 47$, **c** $p = 24$, **d** $g = 28$, **e** $u = 34$, **f** $c = 143$, **g** $m = 112$, **h** $a = 53$, **i** $r = 39$, **j** $k = 136$, **k** $f = 117$, **l** $y = 76$, **m** $w = 52$, **n** $n = 115$, **o** $z = 49$, **p** $s = 117$, **q** $q = 51$, **r** $n = 64$, **s** $v = 48$, **t** $r = 68$, **u** $c = 21$, **v** $j = 59$, **w** $t = 74$, **x** $u = 76$

2 **a** $\angle FTL = \angle MTG$ (vertically opposite $\angle$s)

b Let $\angle FTL = \alpha$
$\angle MTG = \angle FTL$ (proven above)
$= \alpha$
$\angle FET = \angle FTL$ ($\angle$ in alternate segment)
$= \alpha$
$\angle GHT = \angle MTG$ ($\angle$ in alternate segment)
$= \alpha$
$\angle FET = \angle GHT$ (both equal to α)
$\therefore EF \parallel GH$ (alternate $\angle$s are equal)

3 $AB = 2BM$ (line from centre perpendicular to a chord bisects the chord)
$BC = 2BN$ (line from centre perpendicular to a chord bisects the chord)
$BM = BN$ (given)
$\therefore AB = BC$ (equal chords are equidistant from centre)

4 $MP = MQ$ (line from centre perpendicular to a chord bisects the chord)
$MR = MS$ (line from centre perpendicular to a chord bisects the chord)
$\therefore MP - MR = MQ - MS$ (by subtraction)
$\therefore PR = SQ$

5 Let $\angle BAD = \alpha$
$\angle BCD = \angle BAD$ ($\angle$s at circumference are equal)
$= \alpha$
$\angle ABC = \angle BCD$ (alternate $\angle$s, $AB \parallel CD$)
$= \alpha$
$\angle BAD = \angle BCD$ (both equal to α)
$\therefore \Delta APB$ is isosceles

6 $\angle AMO = 90°$ ($\angle$ in a semi-circle)
$AM = MB$ (line from centre perpendicular to a chord bisects the chord)
$\therefore M$ is the midpoint of AB

7 $\angle UTQ = 90°$ (tangent is perpendicular to a radius)
$\angle TVS = 90°$ (co-interior $\angle$s, $PQ \parallel RS$)
$RV = VS$ (line from centre perpendicular to a chord bisects the chord)
$\therefore TU$ bisects RS

8 **a** Let $\angle CAB = \alpha$
$\angle CDB = \angle CAB$ ($\angle$s at circumference are equal)
$= \alpha$
$\angle ACD = \angle CAB$ (base $\angle$s of isosceles Δ, $PA = PC$)
$= \alpha$
$\angle ABD = \angle ACD$ ($\angle$s at circumference are equal)
$= \alpha$
$\angle CDB = \angle ABD$ (both equal to α)
$\therefore \Delta DPB$ is isosceles

b $PA = PC$ (given)
$PB = PD$ (equal sides lie opposite equal $\angle$s)
$\therefore PA + PB = PC + PD$ (by addition)
$\therefore AB = CD$

c $\angle CAB = \angle ABD$ (both equal to α)
$\therefore AC \parallel DB$ (alternate $\angle$s are equal)

9 Let $\angle QTS = \alpha$
$\angle QPS = \angle QTS$ ($\angle$s at circumference are equal)
$= \alpha$
$\angle PSR = \angle QPS$ (alternate $\angle$s, $PQ \parallel RS$)
$= \alpha$
$\therefore \angle QTS = \angle PSR$ (both equal to α)

10 Construct AQ, PQ, QB
$\angle PQA = 90°$ ($\angle$ in a semi-circle)
$\angle PQB = 90°$ ($\angle$ in a semi-circle)
$\angle AQB = 90° + 90°$
$= 180°$
$\therefore A$, Q, B are collinear

11 Let $\angle PAC = \alpha$
$\angle PDB = \angle PAC$ (ext. $\angle$ of a cyclic quadrilateral)
$= \alpha$
$\angle PCA = \angle PAC$ (base $\angle$s of isosceles Δ, $PA = PC$)
$= \alpha$
$\angle PBD = \angle PCA$ (exterior $\angle$ of a cyclic quad.)
$= \alpha$
$\angle PBD = \angle PDB$ (both equal to α)
$\therefore PB = PD$ (equal sides lie opposite equal $\angle$s)
$PA = PC$ (given)
$\therefore PB - PA = PD - PC$ (by subtraction)
$\therefore AB = CD$

12 Let $\angle XWZ = \alpha$
$\angle WXT = \angle XWZ$ (base $\angle$s of isosceles Δ, $TW = TX$)
$= \alpha$
$\angle XTZ = 2\alpha$ (ext. $\angle$ of ΔWTX)
$\angle XYZ = \angle XWZ$ ($\angle$s at circumference are equal)
$= \alpha$
$\angle XOZ = 2\alpha$ ($\angle$ at centre is twice the $\angle$ at circumference)
$\therefore \angle XOZ = \angle XTZ$ (both equal to 2α)

13 Construct XY
Let $\angle CAX = \alpha$
$\angle XYD = \alpha$ (exterior angle of a cyclic quadrilateral)
$\angle XBD = 180° - \alpha$ (opposite $\angle$s of a cyclic quadrilateral are supplementary)
$\angle CAX + \angle XBD = \alpha + (180° - \alpha)$
$= 180°$
$\therefore AC \parallel BD$ (co-interior $\angle$s are supplementary)

14 a Let $\angle CBD = \alpha$
$\angle EFG = \angle CBD$ (given)
$= \alpha$
$\angle EBA = \angle EFG$ (exterior $\angle$ of a cyclic quadrilateral)
$= \alpha$
$\therefore \angle CBD = \angle EBA$ (both equal to α)
b $\angle CED = \angle CBD$ ($\angle$'s at circumference are equal)
$= \alpha$
$\angle ADE = \angle CED$ (both equal to α)
$\therefore PD = PE$ (equal sides lie opposite equal $\angle$s)

15 a Let $\angle BAQ = \alpha$
$\angle ACB = \angle BAQ$ ($\angle$ in alternate segment)
$= \alpha$
$\angle CAB = \angle ACB$ (base $\angle$s of isosceles Δ, $AB = BC$)
$= \alpha$
$\therefore \angle CAB = \angle BAQ$ (both equal to α)
$\therefore AB$ bisects $\angle CAQ$
b $\angle ADB = \angle BAQ$ ($\angle$ in alternate segment)
$= \alpha$
$\angle BDC = \angle CAB$ ($\angle$s at circumference are equal)
$= \alpha$
$\therefore \angle ADB = \angle BDC$ (both equal to α)
$\therefore DB$ bisects $\angle ADC$

16 Let $\angle RTQ = \alpha$ and $\angle QTP = \beta$
$\angle STR = \angle RTQ$ (RT bisects $\angle STQ$)
$= \alpha$
$\angle TSR = \angle QTP$ ($\angle$ in alternate segment)
$= \beta$
$\angle TRP = \alpha + \beta$ (exterior $\angle$ of ΔRST)
$\angle RTP = \alpha + \beta$ (by addition)
$\therefore \angle TRP = \angle RTP$ (both equal to $\alpha + \beta$)
$\therefore PR = PT$ (equal sides lie opposite equal $\angle$s)

17 Let $\angle ABD = \alpha$
$\angle BDC = \angle ABD$ (alternate $\angle$s, $AB \parallel DC$)
$= \alpha$
$\angle BCD = \angle ABD$ ($\angle$ in alternate segment)
$= \alpha$
$\angle BAD = \angle BDC$ ($\angle$ in alternate segment)
$= \alpha$
$\angle ADB = 180° - 2\alpha$ ($\angle$ sum of ΔABD)
$\angle DBC = 180° - 2\alpha$ ($\angle$ sum of ΔBCD)
$\therefore \angle ADB = \angle DBC$ (both equal to $180° - 2\alpha$)
$\therefore BC \parallel AD$ (alternate $\angle$s are equal)

18 Let $\angle PTR = \alpha$
$\angle TQR = \angle PTR$ ($\angle$ in alternate segment)
$= \alpha$
$\angle QTR = 90°$ ($\angle$ in a semi-circle)
$\angle OQS = 90°$ (tangent is perpendicular to a radius)
$\angle PRT = 90° + \alpha$ (ext. $\angle$ of ΔQRT)
$\angle TQS = \angle TQR + \angle OQS$ (by addition)
$= 90° + \alpha$
$\therefore \angle TQS = \angle PRT$ (both equal to $90° + \alpha$)

19 Let $\angle ATP = \alpha$
$\angle TBA = \angle ATP$ ($\angle$ in alternate segment)
$= \alpha$
$\angle BTC = 90° - \alpha$ ($\angle$ sum of ΔBCT)
$\angle ATC = \angle ATP$ (AT bisects $\angle CTP$)
$= \alpha$
$\angle BTA = (90° - \alpha) + \alpha$ (by addition)
$= 90°$
$\therefore AB$ is a diameter ($\angle$ in a semi-circle is 90°)

20 a Let $\angle PTM = \alpha$
$\angle TNP = \angle PTM$ ($\angle$ in alternate segment)
$= \alpha$
$\angle TPN = \angle TNP$ (base $\angle$s of isosceles Δ, $TP = TN$)
$= \alpha$
$\angle TPN = \angle PTM$ (both equal to α)
$\therefore PM = MT$ (equal sides lie opposite equal $\angle$s)

b $\angle TMO = 2\alpha$ (ext. $\angle$ of ΔPTM)
$\angle PTO = 90°$ (tangent is perpendicular to a radius)
$\angle MTO = 90° - \alpha$ (by subtraction)
$OM = OT$ (equal radii)
$\therefore \angle TMO = \angle MTO$ (base $\angle$'s of isosceles Δ)
$\therefore 2\alpha = 90° - \alpha$
$\therefore \alpha = 30°$
$\therefore \angle TMO = \angle MTO = 60°$
$\angle MOT = 60°$ ($\angle$ sum of ΔMOT)
$\therefore \Delta MOT$ is equilateral (all $\angle$s are 60°)

ANSWERS

21 Construct TA and TB
Let $\angle TAP = \alpha$ and $\angle TBP = \beta$
$PA = PT = PB$ (tangents from an exterior point are equal)
$\angle ATP = \angle TAP$ (base $\angle$s of isosceles Δ, $PA = PT$)
$= \alpha$
$\angle BTP = \angle TBP$ (base $\angle$s of isosceles Δ, $PB = PT$)
$= \beta$
$2\alpha + 2\beta = 180°$ ($\angle$ sum of ΔABT)
$\therefore \alpha + \beta = 90°$
$\therefore \angle ATB = 90°$
$\therefore A, B, T$ lie on a semi-circle with diameter AB
$\therefore A, B, T$ are concyclic points

22 Let $\angle APC = \alpha$ and $\angle ATP = \beta$
$\angle TPC = \angle APC$ (CP bisects $\angle BPT$)
$= \alpha$
$\angle TBP = \angle ATP$ ($\angle$ in alternate segment)
$= \beta$
$\angle TDC = \alpha + \beta$ (exterior $\angle$ of ΔTDP)
$\angle TCD = \alpha + \beta$ (exterior $\angle$ of ΔBCP)
$\angle TDC = \angle TCD$ (both equal to $\alpha + \beta$)
$\therefore TC = TD$ (equal sides lie opposite equal $\angle$s)
$\therefore \Delta TCD$ is isosceles

Chapter 14 Review

1 **a** radius, **b** diameter, **c** chord, **d** secant, **e** tangent, **f** sector, **g** segment, **h** arc **2** **a** equal radii, **b** isosceles, **c** $\angle OAB$, $\angle OBA$ **3** circles with the same centre **4** no; O, P, Q, R are not concyclic points **5** **a** 42°, **b** 4 cm, **c** 3 cm, **d** 10 cm, **e** 24 cm, **f** 7 cm **6** **a** $j = 59$, **b** $p = 90$, **c** $c = 37$ **7** **a** $u = 55$, $v = 70$, **b** $e = 136$, $f = 113$, **c** $g = 34$, $h = 38$, **d** $j = 15$, $k = 9$ **8** **a** $k = 90$, **b** $y = 64$, $z = 9$, **c** $p = 63$, $q = 59$ **9** **a** $x = 15$, **b** $p = 6$, $q = 4.5$, **c** $n = 10$, **d** $t = 5$, **e** $u = 4$, **f** $k = 4$ **10** **a** $n = 56$, **b** $e = 41$, **c** $g = 114$, **d** $m = 84$, **e** $u = 13$, **f** $r = 46$

11 **a** Let $\angle QRS = \alpha$
$\angle PQR = \angle QRS$ (alternate $\angle$s, $PQ \parallel RS$)
$= \alpha$
$\angle PSR = \angle PQR$ ($\angle$s at circumference are equal)
$= \alpha$
Now, $\angle QRS = \angle PSR$ (both equal to α)
$\therefore \Delta RST$ is isosceles

b Let $\angle RTS = \alpha$ and $\angle RST = \beta$
$\angle TRP = \alpha + \beta$ (exterior Δ of ΔRST)
$\angle PTR = \angle TRP$ (base $\angle$s of isosceles Δ, $PT = PR$)
$= \alpha + \beta$
$\angle PTQ = \angle RST$ ($\angle$ in alternate segment)
$= \beta$
$\angle QTR = \angle PTR - \angle PTQ$ (by subtraction)
$= \alpha$
Now, $\angle QTR = \angle RTS$ (both equal to α)
$\therefore TR$ bisects $\angle QTS$

c **i** Let $\angle ADC = \alpha$
$\angle AOC = 2\alpha$ ($\angle$ at centre is twice $\angle$ at circumference)
$\angle ABC = 180° - \alpha$ (opposite $\angle$s in a cyclic quadrilateral)
$\angle A = \alpha$ (co-interior $\angle$s, $BC \parallel AO$)
Now, $\angle AOC = 2\alpha$ and $\angle OAB = \alpha$
$\therefore \angle AOC = 2\angle AOB$
ii $\angle AOC + \angle AOB = 180°$ (co-interior $\angle$s, $AB \parallel OC$)
$\therefore 2\alpha + \alpha = 180°$
$\therefore \alpha = 60°$
$\therefore \angle AOC = 120°$

d **i** Let $\angle PST = \alpha$
$\angle PTS = \angle PST$ (tangents from an exterior point)
$= \alpha$
$\angle QTS = \angle PTS$ (TS bisects $\angle QTP$)
$= \alpha$
Now, $\angle QTS = \angle PST$ (both equal to α)
$\therefore TQ \parallel PS$ (alternate $\angle$s are equal)
ii $\angle TQS = \angle PST$ ($\angle$ in alternate segment)
Now, $\angle TQS = \angle QTS$ (both equal to α)
$\therefore \Delta QST$ is isosceles